AD-A111 171 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 22/1
DECENTRALIZED CONTROL OF LARGE SPACE STRUCTURES.(U)
DEC 81 W T MILLER
UNCLASSIFIED AFIT/GAE/AA/81D-20 NL

1 OF 1
AD
A111171

END
DATE
FILMED
8-82
DTIC

LEVEL II

UNITED STATES AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GAE/AA/81D-20

Decentralized Control

of

Large Space Structures

Thesis

AFIT/GAE/AA/81D-20

William T Miller
Capt USAF

AFIT/GAE/AA/81D-20

DECENTRALIZED CONTROL OF LARGE SPACE STRUCTURES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirement for the Degree of

Master of Science

by

William T. Miller

Captain USAF

Graduate Aeronautical Engineering

December 1981

Approved for public release; distribution unlimited.

Preface

The scope of this investigation would have been greatly reduced if it had not been for the ever constant aid and guidance of my thesis advisor, Dr. R. A. Calico. The foundation which he provided made all of the development contained in this research paper possible. For the refinement and technical polishing, I would like to thank Captain J. Silverthorn. I would also like to express my gratitude to the entire department for the courses in control and optimazation which played a key role in the deeper understanding of the theory behind the design. Finally, I would like to acknowledge the support from my wife, Deb, which helped provide motivation throughout this research.

William T. Miller

Contents

Contents (continued)

List of Figures

List of Tables

Abstract

A development and analysis of a single controller, before and after the elimination of "spillover" terms, is implemented to attempt to achieve desired response characteristics of the structure under evaluation. Using this derived data as a basis for comparison, a pair of decentralized controllers are implemented on the structure. The characteristics of the structural response is dramatically improved through the implementation of these decentralized controllers. Problems encountered with the implementation of more than two decentralized controllers are investigated.

The structure used in the controller evaluation is a lumped mass tetrahedron. The four masses of this model are connected through isotropic massless rods capable of supporting axial loading only (no bending). NASTRAN is used to develop a normal mode approximation of the structure, while providing mode shape and frequencies for the resultant twelve mode model. Pointing accuracy of the apex is used in determining figures of merit to evaluate the effectiveness of the control applied. Control is applied through each of the 6 sensor/actuator pairs located on the model.

Controllers are developed using linear optimal techniques which produce feedback gains proportional to the state. The state is represented as modal amplitudes and velocities.

The feedback gains are established via steady state optimal regulator theory. The system response is evaluated initially using only a single controller on an eight mode truncated model then on the entire twelve mode model. A comparison is made with the system prior to the elimination of the observation spillover and after the transformation technique is applied. For the study, four modes are designated as controlled and four as suppressed. The remaining modeled modes are designated residual. An additional controller is added with no addition of sensors or actuators. While the response of the single controller system is unable to meet the design criteria, the addition of a decentralized controller more than adequately achieves the desired response.

The modes designated as residual show very little movement as a result of any of the control forces required or transformations applied to the various systems. As a result of the choice of the higher frequency, modes as residual is verified.

Introduction

With the success of the Space Shuttle Program, we have entered an era where the construction of large space structures will become a reality. To achieve practicality and useful system efficiencies, the proposed sizes of these structures are hundreds of meters in diameter. As the size and flexibility of these structures increase, the number of low frequency structural modes that enter the bandwidth of system controllers also increases. To accomplish control of such vehicles, modeling becomes very critical. Even with improved modeling techniques, there are still modeling inaccuracies which, in the limit, could result in unstable conditions if not properly compensated.

The method of control that is both realizable and viable is modern state space control theory. Using this method, however, due to computational requirements, only a limited number of structural modes can be handled by any single controller. Hence, reduced order controllers are required. The coupling of these reduced order controllers with detailed finite element analysis of the particular structure can be successfully adapted to meet the requirements of several missions and varied flexible structures.

The limiting factor, as to how many of the finite number of modeled modes may be successfully controlled, is

the capabilities of the on-board computer. As a result of these limitations, only these modes which are deemed detrimental to mission requirements are controlled. A specific example would be a photographic satellite where pointing accuracy is considered critical while minor vertical vibrations may be considered inconsequential; as a result, only those modes affecting pointing accuracy would be controlled.

While specific control of these isolated modes would be ideal, it must be realized that in the real situation, the sensor data will be contaminated by the uncontrolled modes and these same uncontrolled modes may be affected by required inputs to the desired modes. These coupling affects are referred to by Balas (Ref 1) as "observation spillover" and "control spillover". It is shown that either of these system coupling effects may lead to overall system instability. The method of control proposed by Balas is based on the use of narrow bandpass filters which effectively comb out the suppressed modes, thus eliminating observation spillover.

Another technique which was first presented by Sesak (Ref 2) and later expanded on by Coradetti (Ref 3) involves a "singular perturbation" technique. It is concluded that this approach, with infinite penalty on spillover, is essentially the same as finding transformation matrix. By applying this transformation matrix to the associated gain matricies, either controller or observer, the spillover terms

would be driven to zero. This method can be effective in removing destabilizing cross coupling terms even if these terms do not result in overall system instability; thus improving system response. These goals are accomplished through the application of state space control techniques coupled with singular value decomposition of rectangular matrices of modal amplitude (Ref 4).

The primary thrust of this investigation is to study the application of the above techniques on the implementation of two or more decentralized controllers on a lumped mass model of a tetrahedron. The primary means of evaluating the effectiveness of the system will be an eigenvalue analysis of the closed loop system and a time response of the pointing angles to initial conditions. This work first investigates all of the results of Janiszewski (Ref 5) and then expands from the single controller model utilizing only eight modes to the multiple controller system using a twelve mode model representation. The elimination of any spillover terms will be accomplished through the implementation of the transformation technique mentioned earlier.

The specifics of the system model used in this investigation will be fully explained in the following section. The model is configured with sensor actuator pairs. The sensors are position sensors only and are used to determine the modal

amplitudes at a point. A singular value decomposition is performed on the matrices of modal amplitudes to obtain a transformation matrix which is employed to eliminate spillover terms. With the addition of a second controller, the improvement of response in the structure is dramatic. Finally, the possibility of implementing more than two decentralized controllers is examined.

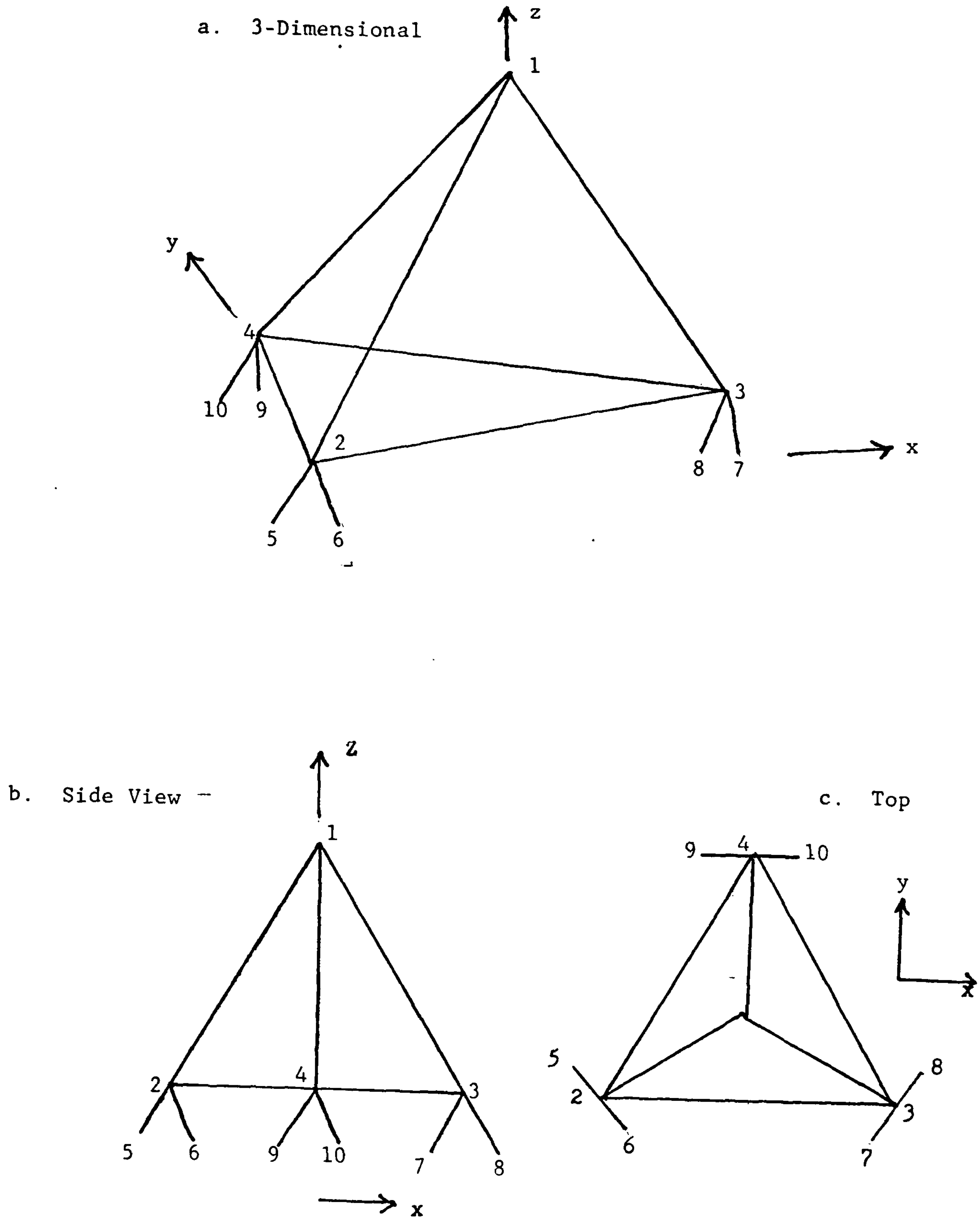
System Model

The model used for this investigation is the tetrahedral model developed at the Charles Stark Draper Laboratory, Inc. This model was arrived at due to the fact that it not only displayed many of the characteristic responses observed in large space structures. It also provided a low order model upon which various control systems could be easily applied so evaluation is simple as a result of the small number of modes present. The performance criteria of the model is based on the motion of the structure at node 1. This is analogous to a line of sight evaluation of a typical optical system.

The finite element model of the structure is displayed in Figure 1. The structure is pin connected at each of the nodes; as a result, it is only capable of transmitting axial forces. A Youngs modulus value of one was used to simplify the stiffness computation. The beams are considered massless with all mass located at nodes 1 through 4. The measured location of each node is listed in Table I.

An eigenvalue analysis of the structure was accomplished on the NASTRAN Computer Program. The key results of the analysis are listed in Table II. The associated eigenvetors are listed in Appendix A. Table III is a listing of the initial conditions that were applied to the model to achieve

Figure 1


a. 3-Dimensional
z
1
y
4
3
10
9
2
8
7
x
5
6
b. Side View
Z
1
2
4
3
5
6
9
10
7
8
x
c. Top
9
4
10
y
x
5
2
8
3
6
7

a time history of the response. For the purpose of this investigation, it is assumed that these values are applied to achieve a desired pointing requirement. As a result, all of these values could be inputs to the controller prior to actuation, thus achieving initial conditions on all of the error terms of zero. The development of these error terms will be covered in the following derivations of the equations of motion.

Table I

Node Coordinates

Mode	X	Y	Z
1	0.0	0.0	10.165
2	-5.0	-2.887	2.0
3	5.0	-2.887	2.0
4	0.0	5.7735	2.0
5	-6.0	-1.1547	0.0
6	-4.0	-4.6188	0.0
7	4.0	-4.6188	0.0
8	6.0	-1.1547	0.0
9	2.0	5.7735	0.0
10	-2.0	5.7735	0.0

Table II

Results of NASTRAN Analysis

Mode	Generalized Mass	Generalized Stiffness	$W_n\left(\frac{RAD}{SEC}\right)$	$\omega\left(\frac{RAD}{SEC}\right)^2$
1	1.0	1.37	1.171	1.37
2	1.0	2.15	1.467	2.15
3	1.0	8.79	2.965	8.79
4	1.0	12.6	3.558	12.6
5	1.0	14.8	3.848	14.8
6	1.0	26.5	5.149	26.5
7	1.0	32.2	5.676	32.2
8	1.0	32.6	5.711	32.6
9	1.0	79.9	8.940	79.9
10	1.0	106	10.030	106
11	1.0	119	10.923	119
12	1.0	195	13.966	195

Table III

Initial Conditions

Mode	Displacement (η)	Velocity ($\dot{\eta}$)
1	-.001	-.003
2	.006	.010
3	.001	.030
4	-.009	-.020
5	.008	.020
6	-.001	-.020
7	-.002	-.003
8	.002	.004
9	.000	.000
10	.000	.000
11	.000	.000
12	.000	.000

Equations of Motion

The equations of motion for the vibrational motion of a large space structure can be written as:

$$M \ddot{\underline{g}} + E \dot{\underline{g}} + K \underline{g} = D \underline{u} \tag{1}$$

where g is an n-vector of generalized coordinates, M is an n x n summetric mass matrix, K is an n x n symmetric stiffness matrix, u is an m-vector of inputs, D is an n x m matrix of modal amplituder evaluated actuator locations, and E is an n x n dumping matrix.

Rewriting equation (1) in a modal coordinates

$$\ddot{\eta} + 2 \xi \omega \dot{\eta} + \omega^2 \eta = \phi^T D \underline{u} \tag{2}$$

where

$$\underline{g} = \underline{\phi} \eta \tag{3}$$

and ϕ^T is the transpose of the n x n model matrix for equation (1). The model matrix ϕ is such that

$$\underline{\phi}^T M \underline{\phi} = \left[I_{\diagdown} \right]$$

$$\underline{\phi}^T K \underline{\phi} = \left[\omega^2_{\diagdown} \right]$$

$$\underline{\phi}^T E \underline{\phi} = \left[2\xi \ \omega_{\diagdown} \right]$$

where all matrices which are displayed are n x n diagonal. To be more explicit, $\left[I_{\diagdown} \right]$ is the identity matrix, $\left[\omega^2 \right]$ is a matrix

of the eigenvalues of equation (1) and $\left[2\zeta\omega\right]$ is the associated damping matrix.

By placing equation (2) into state vector format, we arrive at equation (4):

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \tag{4}$$

where

$$\underline{x}^T = \left[\underline{\eta}^T \;\; \dot{\underline{\eta}}^T\right] \qquad A = \left[\begin{array}{c|c} 0 & I \\ \hline -\omega^2 & -2\zeta\omega \end{array}\right]$$

$$B = \left[\begin{array}{c} 0 \\ \hline \emptyset^T D \end{array}\right]$$

In the practical case, the complete state vector is not available and quation (4) must be supplemented by an output equation. If we assume both position and velcoity information is available, the general output equation becomes:

$$\bar{Y} = C_p\, g + C_v\, \dot{g} \tag{5}$$

in state vector from

$$\bar{Y} = C\,\bar{x} \tag{6}$$

where

$$C = \left[C_p\, \emptyset \;.\; C_v\, \emptyset\right]$$

Equations (4) and (6) are the model of the satellite available to the control designer.

Reduced Order Model

The state vector $\underline{x}$ from above is a 2n-vector that represents the entire structural model. This state vector can be broken into a number of more specific portions in the form

$$\bar{x} = \left[\bar{x}_c^T, \bar{x}_s^T, \bar{x}_r^T, \bar{x}_{um}^T\right]^T \tag{7}$$

$\bar{x}_c \longrightarrow 2\ n_c$ - vector, Controlled modes

$\bar{x}_s \longrightarrow 2\ n_s$ - vector, Suppressed modes

$\bar{x}_r \longrightarrow 2\ n_r$ - vector, Residual modes

$\bar{x}_{um} \longrightarrow 2\ n_{um}$ - vector, Unmodeled modes

The unmodeled modes are mentioned here, but will not appear in any of the derivations. These are modes which are beyond the capability of the mathematical system model to approximate. The unmodeled modes, while they may exist in nature, have not been taken into the modeling effort of the system. The response of the actual system will determine if a more indepth model is required to mathematically approximate the real world system. The controlled modes are those modes determined to require active control to achieve desired system response characteristics. In general, these will not necessarily be the lowest frequency modes.

Due to computational limitations in control of the vehicle, only a small subset of the modeled modes may be con-

trolled, as a result, spillover due to the remaining modes will occur. To eliminate the dilatorions effects of this spillover, a portion of the modeled but uncontrolled mode will be suppressed. The number of which can be suppresed is again dependent on hardware limitations. The remaining modeled modes will be termed residual modes. These residual modes will have "spillover" terms and can be considered representative of those higher frequency modes that are unmodeled.

With these definitions, the system state may be represented by the following equations

$$\dot{\bar{x}}_c = A_c \bar{x}_c + B_c \bar{U} \tag{8}$$

$$\dot{\bar{x}}_s = A_s \bar{x}_s + B_s \bar{U} \tag{9}$$

$$\dot{\bar{x}}_r = A_r \bar{x}_r + B_r \bar{U} \tag{10}$$

$$\bar{Y} = C_c \bar{x}_c + C_s \bar{x}_s + C_r \bar{x}_r \tag{11}$$

The system parameters matrices A, B, and C are as previously described in the text. Furthermore, the matrices B_c can be expressed as: $B_c = \begin{bmatrix} 0 \\ \hline \emptyset_c^T D \end{bmatrix}$

where:

$$\emptyset_c^T D = \begin{bmatrix} a\,(\xi_1) & \cdots\cdots\cdots & a_1\,(\xi\, n_a) \\ \vdots & & \vdots \\ a_{n_c}(\xi_1) & \cdots\cdots\cdots\cdots & a_{nc}\,(\xi\, n_a) \end{bmatrix}$$

where ξ_i is the location of the j^{th} actuator and n_a is the number of actuators and n_c is the number of controlled modes. In this representation, the coefficient matrix, D, has already been multiplied through. Those values for the structure studied are the NASTRAN results in Appendix A. Similar representations of $\emptyset^T D$ can be developed for both the suppressed and residual modes with the only difference occurring as n_s and n_r, the number of suppressed and residual modes, respec= tively are used in place of n_c.

In a similar manner, if we assume only point sensors located at the points ξ_i then

$$C_c = C_{p_c} \emptyset$$

where

$$C_{p_c}\emptyset = \begin{bmatrix} a_1(\xi_1) \ldots\ldots\ldots\ldots a_{nc}(\xi_1) \\ \\ a_1(\xi_{n_{sen}}) \ldots\ldots\ldots a_{nc}(\xi_{n_{sen}}) \end{bmatrix}$$

where n_{sen} is the number of sensors employed.

The equations developed to this point are quite general and independent of structural complexity. With increased complexity, the sizes of the respective matrices are the only variables that will increase in dimension.

The general nature of the development is the key to

its wide area of possible application to a variety of large space structures.

Modal Control

The control modal upon which the control design is to be based is given by

$$\dot{\bar{X}}_c = A_c \bar{x}_c + B_c \bar{U} \tag{8}$$

$$\bar{Y} = C_c \bar{x}_c + C_s \bar{X}_s + C_r \bar{X}_r \tag{11}$$

Three feedback controllers are examined. The form of control for each controller is:

$$\bar{U}_1 = G \hat{X}_c \tag{12}$$

$$\bar{U}_2 = G_1 \hat{X}_1 + G_2 \hat{X}_2 \tag{13}$$

$$\bar{U}_3 = G_1 \hat{X}_1 + G_2 \hat{X}_2 + G_3 \hat{X}_3 \tag{14}$$

where $\hat{X}_1$, $\hat{X}_2$, and $\hat{X}_3$ are the specific modes controlled by each controller. Ideally the control law would be $G \bar{X}$ but in this case, where not all of the states are available, the estimated values of the states must be used. The individual closed loop system matrices will be developed sequentially in the following discussion.

Single Controller

Since we are unable to directly measure the entire

state vector, it is necessary to employ an observer of the form:

$$\dot{\hat{X}}_c = A_c \hat{X}_c + B_c U + K(y - \hat{y}) \tag{15}$$

$$\hat{Y} = C_c \hat{X}_c \tag{16}$$

where

$\hat{X}_c$ · estimated state vector

$\hat{Y}_c$ · estimated output vector

The observer gain matrix is chosen such that the error in the state estimate, represented by

$$\bar{e}_c = \hat{X}_c - \bar{X}_c \tag{17}$$

is asymptotically stable.

The closed loop system stability, including controller and observer, can be evaluated by writing the state equations for an augmented state vector defined below. For the single controller $\underline{Z}$ will be defined as follows:

$$\underline{Z} = (\bar{X}_c^{\,T}, \bar{e}^T, \bar{X}_s^{\,T}, \bar{X}_r^{\,T})^T \tag{18}$$

With the definition the overall closed loop system matrix can be represented in block matrix form as:

$$\dot{\bar{Z}}(t) = \begin{bmatrix} A_c + B_c G & B_c G & 0 & 0 \\ 0 & A_c - KC_c & KC_s & KC_r \\ B_s G & B_s G & A_s & 0 \\ B_r G & B_r G & 0 & A_r \end{bmatrix} Z(t) \tag{19}$$

At this point it is of interest to look at the development of the observer gain matrix, K, and the control feedback gain matrix, G. First consider the control gain matrix G.

In order to use linear optimal regulator theory, a performance index is defined as:

$$J = 1/2 \int_0^{\infty} (\bar{X}_c^T Q \bar{X}_c + u^T R u) \, dr \qquad (20)$$

where

Q - is an n x n positive semidefinite weighting matrix.

R - is an m x m positive definite weighting matrix.

This performance index, subject to

$$\dot{\bar{X}}_c = A_c \bar{X}_c + B_c U$$

is minimized with

$$\bar{U} = G \hat{X}_c$$

and

$$G = -R^{-1} B_c^T S \qquad (21)$$

and S is the solution to the matrix Recatti Equation.

$$S A_c + A_c^T S - S B_c R^{-1} B_c^T X + Q = 0 \qquad (22)$$

The development of the observer matrix can be formu-

formulated in an identical development once it is realized that the eigenvalues of the matrix, $(A_c - KC_c)$, are equal to the eigenvalues of the transpose of the matrix. The system can be then written as:

$$\dot{\bar{W}}(t) = A_c^T \bar{W}(t) - C^T g(t)$$

$$g(t) = K^T W(t)$$

Using this system and defining a similar performance index as listed in equation (20) with the substitution of W for X_c leads to the solution for the gains K^T in the form.

$$\bar{K}^T = +\bar{R}_{ob}^{-1} \bar{C}_c \bar{P}$$

where $\bar{P}$ is the solution to the steady state algebraic matrix Ricatti Equation:

$$P A_c^T + A_c P - P C_c^T R_{ob}^{-1} C_c P + Q_{ob} = 0$$

While the system of equations is not block triangular, it can be made block triangular through the elimination of control spillover or observation spillover. Once we have achieved suppression of the appropriate terms, the stability of the system is assured through the proper design of the controller and observer. For the purpose of this research, elimination of observation spillover has been deemed more practical and cost efficient. Additional sensors to achieve the desired observation spillover is much easier to implement

than increasing the number of actuators to achieve spill-over suppression.

Dual Controller

The following development of a two controller system parallels that of the single controller. The control law to be applied is as stated in equation (13). In this system rather than defining a specific number of modes as suppressed, the goal is to achieve two decentralized controllers which will be independent of each other.

The two state equations are:

$$\dot{\bar{X}}_1 = A_1 \bar{X}_1 + B_1 \bar{U} \tag{23}$$

$$\dot{\bar{X}}_2 = A_2 \bar{X}_2 + B_2 \bar{U} \tag{24}$$

Recalling the general observer equation (15) and equation (16) where the control law applied is equation (13).

$$\hat{X}_i = \bar{A}_i \hat{X}_i + \bar{B}_i \bar{U} + K (\bar{y} - \hat{y}) \qquad i = 1, 2$$

$$\hat{Y}_i = C_i \hat{X}_i \qquad i = 1, 2$$

$$\bar{U} = G_1 \hat{X}_1 + G_2 \hat{X}_2$$

The error in each system is described as

$$\bar{e}_1 = \hat{X}_i - \bar{X}_i \qquad i = 1, 2 \tag{25}$$

By applying equations (11), (23), (24), (25), and the estimator equations which are listed above, it can be shown that $\dot{\bar{e}}_i$ is described by:

$$\dot{\bar{e}}_1 = \dot{\hat{X}}_1 - \dot{\bar{X}}_i = (A_1 - K_1 C_1)\bar{e}_1 + K_1 C_2 \bar{X}_2 + K_1 C_r \bar{X}_r \qquad (26)$$

$$\dot{\bar{e}}_1 = \dot{\hat{X}}_2 - \dot{\bar{X}}_1 = (A_2 - K_2 C_2)\bar{e}_2 + K_2 C_1 \bar{X}_1 + K_2 C_r \bar{X}_r \qquad (27)$$

The associated $\dot{\bar{X}}$ equation may be simply derived using the system equation (23) and the control law (13). The resulting equation is:

$$\dot{\bar{X}}_1 = (A_1 + B_1 G_1) \bar{X}_1 + B_1 G_1 \bar{e}_1 + B_1 G_2 \bar{e}_2 + B_1 G_2 \bar{X}_2 \qquad (28)$$

A similar application of the control law and the residual model equation (10) provides the following results:

$$\dot{\bar{X}}_r = A_r \bar{X}_r + B_r G_1 \bar{X}_1 + B_r G_1 \bar{e}_1 + B_r G_2 \bar{X}_2 + B_r G_2 \bar{e}_2 \qquad (29)$$

By defining an overall system vector z of the fom:

$$\bar{Z}^T = \left[\bar{X}_1{}^T, \bar{e}_1{}^T, \bar{X}_2{}^T, \bar{e}_2{}^T, \bar{X}_r{}^T\right] \qquad (30)$$

The closed loop system model including the two decentralized controllers, each utilizing state variable feedback, can be written as:

$$\dot{Z} = \begin{bmatrix} A_1 + B_1 G_1 & B_1 G_1 & B_1 G_2 & B_1 G_2 & 0 \\ 0 & (A_1 - K_1 C_1) & K_1 C_2 & 0 & K_1 C_r \\ B_2 G_1 & B_2 G_1 & (A_2 + B_2 G_2) & B_2 G_2 & 0 \\ K_2 C_1 & 0 & 0 & (A_2 - K_2 C_2) & K_2 C_r \\ B_r G_1 & B_r G_1 & B_r G_2 & B_r G_2 & A_r \end{bmatrix} Z(t) \quad (31)$$

It is apparent that the suppression of all the "observation spillover" or the "control spillover" terms is insufficient to completely triangularize the system even in the absence of residual modes. To achieve a closed loop system with the above characteristics, it is necessary to suppress the control spillover term of one system, e.g., $B_1 G_2$, while suppressing the observation spillover of the other system, $K_1 C_2$. The judicious selection of modes again is critical so as to provide a frequency separation between the lower frequency controller and the residual modes.

The primary advantage of two controllers is the number of modes controlled can be divided between the two systems. This is important since the computational burden of solving the Ricatti Equation increases roughly as the cube of the order of the equation (Ref 3). Therefore, the advantages of solving the Ricatti Equation of two smaller controllers is apparent.

Three Controllers

To avoid a repetition of all of the equations developed in the previous section, it can be stated that the control law of equation (14) was applied to arrive at the closed loop system model of this section. The $\bar{Z}$ vector is defined as:

$$\bar{Z} = \left[\bar{X}_1^T,\ \bar{e}_1^T,\ \bar{X}_2^T,\ \bar{e}_2^T,\ \bar{X}_3^T,\ \bar{e}_3^T,\ \bar{X}_r^T\right]^T \tag{32}$$

This results in an overall closed loop system equation:

$$\dot{\bar{Z}}(t) = \begin{bmatrix} A_1+B_1G_1 & B_1G_1 & B_1G_2 & B_1G_2 & B_1G_3 & B_1G_3 & 0 \\ 0 & A_1-K_1C_1 & K_1C_2 & 0 & K_1G_3 & 0 & K_1C_r \\ B_2G_1 & B_2G_1 & A_2+B_2G_2 & B_2G_2 & B_2G_3 & B_2G_3 & 0 \\ K_2C_1 & 0 & 0 & A_2-K_2C_2 & K_2C_3 & 0 & K_2C_r \\ B_3G_1 & B_3G_1 & B_3G_2 & B_3G_2 & A_3B_3G_3 & B_3G_3 & 0 \\ K_3C_1 & 0 & 0 & 0 & 0 & A_3-K_3C_3 & K_3C_r \\ B_rG_1 & B_rG_1 & B_rG_2 & B_rG_3 & B_rG_3 & B_rG_3 & A_r \end{bmatrix} \tag{33}$$

As discussed earlier, the system presented here cannot be triangularized through complete elimination of observation spillover or control spillover. There are two approaches that can be utilized in the examination of the three controller system. First, through the judicious positioning of sensors, the modal amplitude matrix and thus the system parameter

matrices, B and C, of on controller can be made orthogonal to the remaining two controllers. To completely decouple the system, the terms which must be eliminated are listed in Table IV. By arranging the modes such that two of the controllers operate on modes such that two of the controllers operate on modes that are orthogonal to the third controller, the system would reduce to a two controller system. The system model used in this study has been determined to contain such properties. This will be specifically demonstrated in the investigation portion of the text. At this point suffice it to say that the twelve modes modeled can be divided into two orthogonal groupings.

As an example, let controllers one and two operate on the first group of orthogonal modes while controller two operates on a portion of the second grouping. As a result, all cross terms between one or two and three will be equal to zero. This will reduce Table IV to

$$B_1G_2 = 0 \qquad\qquad B_2G_1 = 0$$
$$\text{or}$$
$$K_1C_2 = 0 \qquad\qquad K_2C_1 = 0$$

which are the terms required equal to zero to decouple the two controller system, therefore demonstrating the ability to reduce the system to a two controller problem. The second method of system suppression would require an optimization process included in the transformation formation such that

Table IV

Total Decouple of 3 Controller

$B_2G_3 = 0$				$B_2G_1 = 0$
$K_2C_3 = 0$				$K_2C_1 = 0$
$B_1G_3 = 0$		OR		$B_3G_1 = 0$
$K_1C_3 = 0$				$K_3C_1 = 0$
$B_1G_2 = 0$				$B_3G_2 = 0$
$K_1C_2 = 0$				$K_3C_2 = 0$

such terms as B_2G_1 and B_3G_1 are approximately zero. While the possibility of obtaining a transformation matrix orthogonal to both matrices is highly unlikely, an optimazation process can be applied to reduce the value of these spillover terms to insignificant values relative to the system dynamics. This second method would require more on-board computational capabilities which may result in exceeding the designed capacity of the system. As a result, this method would be far more costly to implement thus making the first method the only viable approach. Since it has been demonstrated that the system can always be reduced to a two controller problem through proper sensor and actuator location, only the investigation of the single and dual controllers will be carried out in this research.

Transformation Technique

This section is designed to describe in further detail those methods applied to the model to achieve the block triangular form. This will require the elimination of the cross coupling terms such as K_1C_1 and B_1G_2 in the two controller system. The entire thrust of this method is to drive these terms to zero while keeping the terms B_1G_1, B_2G_2, and K_1C_1, K_2C_2 not equal to zero. This is first done for the single controller case. The technique is then applied to the two controller problem by eliminating the control spillover of one controller while operating on the observation spillover of the second controller.

For the single controller system, the elimination of observation spillover is achieved if a K matrix can be found such that

$$K \quad C_s = 0 \tag{34}$$

$$K \quad C_r = 0 \tag{35}$$

while

$$K \; C_c = 0 \tag{36}$$

The final equation is constraint that must be met in order to maintain observability over the controlled modes.

While it would be optimal to achieve both equation (34) and equation (35) in the system model, in the actual structure this would not be fully realizable. This is primarily

due to the large number of modes that are physically present in the structural model. As a result, only a subset of the modeled modes will be suppressed. Thus, only equation (34) will be satisfied.

The selection of those modes to be designated as suppressed or as residual is somewhat arbitrary and could be established through an iterative process. Those modes you are most interested in suppressing are those modes which, even though stable, are weakly damped and thus may be driven unstable as a result of the observation spillover. The selection of those modes as residual would be best designated as those modes which are actually shifted further to the left of the jω-axis as a result of the observation spillover, thus stabilizing these modes. Another choice, the one used in this investigation, is to suppress all uncontrolled modes below a certain frequency. The primary assumption here is that the higher frequency modes fall outside of the bandwidth of the controller.

However the selection of the suppressed modes is accomplished, the system to be examined is:

$$C_s^T K^T = 0 \tag{37}$$

This is nothing more than the transpose of equation (34), however, this form of equation is more useful as will become apparent.

To achieve this desired result, the K^T matrix of equation (37) must be transformed such that it is orthogonal to the rows of C_s^T (columns of C_s). The C_s^T matrix is sized such that it has the number of columns that corresponds to the number of sensors (n_{sen}) and a non zero row length of the number of suppressed modes (n_s). Looking at the equation of the transformation required

$$C_s^T t = 0 \tag{38}$$

The number of linearly independent algebraic solutions, t, are specified as the difference between the rank of C_s^T and n_{sen} The number of suppressed modes is equal to or greater than the number of sensors, no solution vector t can be found unless the rows of C_s^T are not linearly independent. As a result of this relation, in general, the number of modes that can be suppressed can not exceed the number of sensors available. In terms of output we will define v by:

$$v = T \bar{y} \tag{39}$$

Where T is matrix whose rows are composed of the solution vectors t. Substituting for the value of y:

$$v = T C_c X_c + T C_r X_r + T C_s X_s \tag{40}$$

however:

$$T C_s = 0 \tag{41}$$

As a result of the output, v does not contain the suppressed modes. The new control problem to be considered can be stated as:

$$\dot{\bar{X}}_c = A_c \bar{X}_c + B_c U \tag{42}$$

$$v = T C_c X_c + T C_r X_r = C_c^* X_c + C_r^* X_r \tag{42b}$$

The output v is no longer a vector of dimension n_{sen} but has dimension (n_{sen}- Rank of C_s^T). The suppression may, therefore, be thought of as replacing a system of n_{sen} sensors with n_{sen}-r) synthetic sensors.

As long as the system of equations (42) are observable and controllable, the stable matrices $A_c + B_c G$ and $A_c - K C_c^*$ can be formed and placed in the overall system matrix of equation (19) in which the observation spillover will have been removed. If the suppressed modes for this system are properly chosen, the entire system will remain stable.

With that general overview of the purpose and result of the technique, the specifics of obtaining the matrix T will be developed. The matrix of interest in this technique of observation suppression is C_s. This matrix can be written in the form:

$$C_s = W \xi V^T \tag{43}$$

where:

W is an $n_{sen} \times n_{sen}$ orthogonal matrix of left singular vectors.

V is an $n_s \times n_s$ orthogonal matrix of right singular vectors.

and

$$\Sigma = \left[\begin{array}{c:c} S & 0 \\ \hdashline 0 & 0 \end{array}\right] \qquad (44)$$

such that S is a r x r matrix of the non-zero singular values of C_s and r is the rank of C_s as previously stated. Furthermore, the matrix W can be partitioned such that:

$$W = \left[W_r \,\vdots\, W_q\right] \qquad (45)$$

The partition W_r is an n_{sen} x r matrix of left singular vectors associated with the non-zero singular values of C_s and W_q is an n_{sen} x q matrix of left singular vectors associated with the zero singular values.

Since W is an orthogonal matrix, the product of W_q^T and W_r is zero which leads to

$$W_q^T C_s = W_q^T W_r S V_r^T = 0$$

As a result, it is obvious that the T matrix sought is composed of the left singular vectors associated with the

zero singular values of C_s. This transformation is applied to the system as specified in equations (42).

Computer Model

The primary goals in the formulation of the program were flexibility and simplicity. The program is capable of making several diverse runs depending on the desired output or the particular area of interest being examined. The program generates output data for a single controller or a dual controller model. In either of these types of runs, the inclusion of the residual modes is optional.

The repetitive nature of the formation of the parameter matrices (A, B, C,); in the control, suppressed, and residual form; led to these being structured in a subroutine. This format also added the flexibility to change the size of the matrices to meet specific requirements of various investigations. The formation of the initial condition vector is also accomplished in a subroutine. There is a separate subroutine for the different initial condition vectors required for the single or dual controller model.

For program initialization certain data is required from either permanent files or parameter assignments. Required data from the user is the number of controlled, suppressed, and residual modes followed by the number of actuators and sensors. Finally, the damping ratio for each of the modes, assumed equal, must be designated; 0.005 for the system studied. The system will then read from permanent file the NASTRAN values of the modal amplitude at each actuator location

The same data is then entered for each of the sensor locations. For the model considered these values are identical since the sensors and actuators are colocated, however, it was considered necessary to make separate entires to accommodate those possible situations where the sensors and actuators are not colocated. Finally, the associated frequency for each of the modes is read in from permanent file. For time response calculation the initial conditions for the system and the mode shapes of the point of interest msut also be made available.

With the preload of this data, the modal arrangement as controlled, suppressed or residual is at the option of the operator. The various modes may be moved in any manner desired by the operator without the requirement for the preload of any additional modal information. Once a particular selection is made, the program will form the specified matrices and the associated initial condition vector. With the system now completely structured, the steady state feedback matrices are formed (G and K). This is accomplished through the execution of a series of sophisticated subroutines created by Kleinman Ref 6), which provide a numerical solution to the matrix Ricatti Equation. With these matrices formed, an overall system matrix as depicted in either equation (19) or equation (31) is formed dependent on whether a single or dual controller run has been indicated.

At this point one can execute the option to create a time history of the line of sight at point 1 in the x and y directions. This type of system response was used since the pointing accuracy of the vehicle was a criteria for determining the success of the system controller(s). The line of sight was calculated with the use of the zero input equation for the state equation:

$$\dot{\bar{X}} = A \bar{x} + B \bar{u} \tag{46}$$

$$Y = C X \tag{47}$$

The zero input equation is

$$X\ (kdt) = e^{Adt} X \quad (k-1)dt \tag{48}$$

To minimize any problems that might arise as a result of rapid system osciallations not perceived by the discrete model, a DT = .01 was utilized. The e^{Adt} matrix is determined through the Taylor series expansion of the term in the following equation:

$$e^{Adt} = I + Adt + \frac{A^2 dt^2}{2!} + \frac{A^3 dt^3}{3!} + \ . \ . \ . \tag{49}$$

The value of the displacement at position 1, in the x and y direction, is calculated each .5 second up to 20 seconds using the mode shapes, previously loaded, and the computed value of X (t). This displacement is calculated through the summation formula:

$$X_n(t) = \sum_{i=1}^{m} \phi_{ni} X_i(t) \qquad n = 1, 2 \qquad (50)$$

Where m is the number of modes in model. For n = 1, the equation computes the line of sight displacement in the X direction and n = 2 represents the Y direction displacement.

At this point, the eigenvalues of the matrices (A+BG), (A-KC) and the overall system matrix are computed. This analysis was accomplished by implementing the subroutine EIGRF from the International Mathematical and Statistical Library (IMSL). The eigenvalues of the closed loop system matrix, since they reflect the overall systems stability, determining success of the system suppression. Comparison of these eigenvalues with the eigenvalues of (A+BG) and (A-KC) demonstrate which modes were most affected.

The suppression of the system varies whether one or two controllers are implemented by the operator. With a single controller, the observation spillover is eliminated by accomplishing a singular value decomposition of the C_s^T matrix. This is achieved through the execution of the IMSL subroutine LSVDF. Then by using the associated singular vectors, a transformation matrix is generated. The overall system is then recreated using the transformation technique as is described in section IV. Once the new gain matrix is created, the program loops back and initiates another time

response and eigenvalue analysis.

In the case of the two controller system of equation (31), it is apparent that the elimination of observation spillover is insufficient to completely decouple the system in the absence of the residual modes. In this case one must eliminate control spillover of one system while eliminating the observation spillover of the other system. The elimination of system one observation spillover was implemented to take advantage of that part of the computer model already structured. The control spillover are implemented in the overall system and a time response calculation followed by an eigenvalue analysis is then accomplished.

Since there exists the possibility that modes other than suppressed modes are adversely affected, a series of calculations are required to insure that controlled modes are not aligned with any of suppressed modes. A further check to insure that controlled modes are a linear combination of the suppressed modes is accomplished at the end of the run. When one of these cases are encountered, a regrouping of the modes is required to avoid the detrimental affects of the suppression on the overall system response.

While the model used in this study is very specific in its definition, the subroutine structure of the program provides the flexibility to analyze a variety of other system through the simple restructuring of the subroutines that

form the A, B, and C matrices to comply with the new system to be analyzed. The only requirement is that the system can be written in the format:

$$\dot{\bar{X}} = A \tilde{x} + B u$$

and

$$\bar{Y} = C \tilde{x}$$

Investigation

A building block method of research was deemed the best approach to make a thorough study of the complete system. As earlier explained, it was determined that due to hardware and cost considerations, observation spillover elimination would be employed when at all possible. Initially, the basic system was researched using only controlled and suppressed modes. This was done to confirm the fact that the system could be block triangularized through the elimination of observation spillover. The eigenvalue analysis that resulted is displayed in Table V. This analysis was accomplished on the first eight modes of the NASTRAN structural analysis. The control weighting matrix was equal to $Q = 20 [I]$.

At this point, the four residual modes were added to complete the implementation of the twelve mode model. The system was analyzed to determine the effect of the four additional modes on the response characteristics of the system. The eigenvalue analysis of this system is listed in Table VI with the associated time response listed in Table VIa.

The value of Q was left at the value of $20 [I]$ for the remainder of the investigation so that all results could be associated with either the number of controllers implemented or the various groupings of the modes.

Table V

System Eigenvalue Analysis - Single Controller

Modal Assignments

Controlled	Suppressed	Residual
1,2,4,5	3,6,7,8	None

Overall System Eigenvalues

Before Transformation			After Transformation		
-.00070	+	5.73519	-.02837	+	5.76583
-.01995	±	5.75026	-.02855	±	5.71073
-.02574	±	5.14935	-.02574	±	5.14935
-.06602	±	3.55616	-1.04342	±	3.42286
-1.12459	±	3.27924	-.34269	±	1.16273
-.[illegible]5796	±	1.25179	-.01482	±	2.96457
-.[illegible]5138	±	0.77807	-.50461	±	1.43353
-1.[illegible]3795	±	3.76850	-.13144	±	1.46583
-1.28709	±	3.54688	-.18344	±	1.16990
-.03486	±	3.00503	-1.02693	±	3.42569
-.28446	±	1.58838	-1.24619	±	3.66533
-.71752	±	1.00300	-1.26459	±	3.66181

Table Va

Time Response - Single Controller

Modal Assignments

Controlled	Suppressed	Residual
1,2,4,5	3,6,7,8	None

Before Transformation			After Transformation		
Time	Los-X	Los-Y	Time	Lox-X	Los-Y
.5	.003950	.001608	.5	.003957	.001611
1.0	.003446	.000060	1.0	.00337[illegible]	.000068
1.5	.001254	-.000043	1.5	.000914	-.001049
2.0	.000368	-.000536	2.0	-.000466	-.001016
2.5	.000425	.000233	2.5	-.000395	-.000437
3.0	-.000500	.000408	3.0	-.000873	-.000086
3.5	-.001138	.000139	3.5	-.001299	-.000201
4.0	-.000528	.000641	4.0	-.000557	.000467
4.5	.000539	.000541	4.5	.000561	.000633
5.0	.000327	.000243	5.0	.000656	.000686
5.5	-.000408	-.000874	5.5	-.000147	-.000257
6.0	-.000202	-.000622	6.0	-.000256	-.000289
6.5	.000544	-.000322	6.5	.00027[illegible]	-.000141
7.0	.000482	.000094	7.0	.000491	.000068
7.5	-.000412	-.000172	7.5	-.000236	-.000241
8.0	-.000425	.000081	8.0	-.0004[illegible]	-.000369
8.5	.000177	.000614	8.5	.0000[illegible]	.000201
9.0	.000367	.000485	9.0	.0005[illegible]	.000292
9.5	-.000394	.000082	9.5	-.000051	.000201
10.0	-.000469	-.000474	10.0	-.000519	-.000354
10.5	.000206	.000047	10.5	-.00006[illegible]	.000084
11.0	.00063	-.000112	11.0	.000457	.000172
11.5	.000009	-.000109	11.5	.000199	.000124
12.0	-.000402	-.000474	12.0	-.000459	-.000335
12.5	.000077	.000157	12.5	-.00017[illegible]	-.000157
13.0	.000379	.000329	13.0	.00036[illegible]	.000252
13.5	-.000034	.000182	13.5	.000346	.000123
14.0	-.000583	-.000119	14.0	-.00036[illegible]	-.000111
14.5	-.000083	-.000046	14.5	-.000344	-.000287
15.0	.000376	.000317	15.0	.000264	.000270
15.5	.000226	-.000113	15.5	.000393	.000142
16.0	-.000358	-.000186	16.0	-.000186	-.000032
16.5	-.000096	-.000332	16.5	-.000463	-.000320
17.0	.000362	.000299	17.0	.000167	.000111
17.5	.000217	.000008	17.5	.000405	.000238
18.0	-.000333	-.000026	18.0	.000007	-.00038
18.5	-.000322	-.000156	18.5	-.000494	-.000226
19.0	.000263	.000251	19.0	.000004	-.000076
19.5	.000224	.000195	19.5	.000405	.000326
20.0	-.000162	-.000166	20.0	.000129	-.000014

Based on the affect of the modes on the motion of the structure, it was deemed most beneficial to control modes 1, 2, 4, 5 while suppressing modes 3, 6, 7, 8. Finally, the residual modes were 9, 10, 11, 12. The choice of the residual modes was based on the fact that because of freqency separation, these modes would be unaffected by the control that was applied to lower frequency modes. This premise is born out in the analysis of the eigenvalues presented in Table VI. This shows a damping ratio for the residual modes of approximately .005 or greater, since 0.005 was used as the open loop damping ratio, the controller has increased the damping of each of the residual modes even through they were not included in the optimal control formulation.

Table VI

System Eigenvalue Analysis - Single Controller

Modal Assignments

Controlled	Suppressed	Residual
1,2,4,5	3,6,7,8	9,10,11,12

Overall System Eigenvalues

Before Transformation			After Transformation		
-.05653	±	10.34269	-.05548	±	10.34651
- 07015	±	13.96805	-.07001	+	13.96821
-.06235	±	10.04103	-.05902	±	10.94113
-.03573	±	8.95523	-.05060	±	8.94695
-.00077	±	5.73585	-.02837	±	5.67583
-.81824	±	3.67870	-.02855	±	5.71073
-1.25313	±	3.02515	-.82055	±	3.65895
-.56844	±	.74670	-1.21939	±	3.04764
-.15507	±	1.25423	-.37831	±	1.09691
-.02030	±	5.70036	-.17427	±	1.20133
-.02574	±	5.14935	-.02574	±	5.14935
-1.08395	±	3.89998	-1.06686	±	3.89134
-1.42995	±	3.33404	-1.42363	±	3.34893
-.03425	±	3.00435	-.04182	±	2.96457
-.75428	±	.89816	-.51201	±	1.40017
-.26000	±	1.59460	-.13381	±	1.47589

Table VIa

Time Response - Single Controller

Modal Assignments

Controlled	Suppressed	Residual
1,2,4,5	3,6,7,8	9,10,11,12

Before Transformation

Time	Los-X	Los-Y
.5	.004036	.001648
1.0	.003428	.000076
1.5	.001145	-.000911
2.0	.000315	-.000623
2.5	.00497	.000167
3.0	-.000382	.000388
3.5	-.000950	.000183
4.0	-.000289	.000720
4.5	.000717	.000620
5.0	.000350	.000267
5.5	-.000491	-.000893
6.0	-.000328	-.000667
6.5	.000421	-.000363
7.0	.000420	.000080
7.5	-.000379	-.000146
8.0	-.000356	.000113
8.5	.000226	.000623
9.0	.000403	.000490
9.5	-.000396	.000069
10.0	-.000526	-.000505
10.5	.000150	.000019
11.0	.000613	-.000097
11.5	.000038	-.000087
12.0	-.000392	-.000462
12.5	.000092	.000170
13.0	.000404	.000345
13.5	-.000049	.000171
14.0	-.000622	-.000150
14.5	-.000094	-.000057
15.0	.000383	.000318
15.5	.000236	-.000105
16.0	-.000347	-.000179
16.5	-.000082	-.000318
17.0	.000356	.000297
17.5	.000208	.000004
18.0	-.000341	-.000029
18.5	-.000333	-.000162
19.0	.000254	.000246
19.5	.000246	.000202
20.0	-.000139	-.000155

After Transformation

Time	Los-X	Los-Y
.5	.004043	.001651
1.0	.003365	.000063
1.5	.000816	-.001118
2.0	-.000455	-.001141
2.5	-.000390	-.000596
3.0	-.000841	-.000229
3.5	-.001093	-.000274
4.0	-.000361	.000469
4.5	.000710	.000686
5.0	.000671	.000746
5.5	-.000244	-.000269
6.0	-.000423	-.000266
6.5	.000087	-.000141
7.0	.000373	.000079
7.5	-.000255	-.000203
8.0	-.000366	-.000328
8.5	.000178	.000213
9.0	.000639	.000277
9.5	.000009	.000160
10.0	-.000584	-.000424
10.5	-.000179	.000016
11.0	.000379	.000150
11.5	.000178	.000148
12.0	-.000467	-.000281
12.5	-.000119	-.000087
13.0	.000434	.000307
13.5	.000366	.000132
14.0	-.000406	-.000144
14.5	-.000381	-.000334
15.0	.000223	.000223
15.5	.000365	.000113
16.0	-.000168	-.000024
16.5	-.000419	-.000288
17.0	.000195	.000137
17.5	.000415	.000255
18.0	.000008	-.000026
18.5	-.000517	-.000232
19.0	-.000036	-.000198
19.5	.000389	.000310
20.0	.000140	-.000017

Values are shown before and after suppression

The next logical step was to examine the system performance with the dual controller system of equation (31) was implemented. Again, this research was done with Q = 20 [I]. The system was divided such that controller one handled modes 1, 2, 4, and 5, as determined necessary to achieve acceptable pointing accuracies; controller two was initially specified as modes 3, 6, 7, and 8. The two controller model was run agains the eight mode truncated model to confirm the effectiveness of the method of suppression employed. These results can be seen in Table VII.

With these results the additional residual mode (9, 10, 11, 12) were included in the model to check for any adverse effects due to these modes as was encountered in the earlier investigation of the single controller. In this case, the overall system retained the achieved stability as is seen in Table VIII. The desirable results that were achieved in this arrangement were that the system achieved the desired accuracies in the x and y directions within approximately 10 seconds. The associated time response printout to each of the above runs are presented in Tables IV through VIII.

To obtain a more indepth understanding of the modal characteristics of the structure, a thorough study of the

Table VII

System Eigenvalue Analysis - Two Controllers

Modal Assignments

Controller #1	Controller #2	Residual
1,2,4,5	3,6,7,8	None

Overall System Eigenvalues

Before Transformation	After Transformation
-1.66818 ± 5.62098	-.02837 ± 5.67583
-.59552 ± 0.64991	-1.57647 ± 5.46629
-.16098 ± 1.24035	-1.07376 ± 5.57038
-1.36745 ± 5.20773	-1.51740 ± 5.50617
-1.63542 ± 5.66741	-1.61266 ± 4.90964
-.96051 ± 3.62505	-1.61259 ± 4.90962
-1.31648 ± 5.26339	-.34269 ± 1.16723
-1.16440 ± 3.20916	-.50461 ± 1.43353
-1.61266 ± 4.90964	-.13144 ± 1.46588
-1.61259 ± 4.90962	-.18344 ± 1.16990
-.80770 ± 0.74588	1.04342 ± 3.42286
-.30549 ± 1.54735	-.83319 ± 2.39507
-1.21882 ± 3.83849	-.94909 ± 2.84746
-1.35638 ± 3.48462	-1.26459 ± 2.66181
-.90125 ± 2.71050	-1.24619 ± 3.66533
-.92979 ± 2.99683	-1.02693 ± 3.42659

Table VIIa

Time Response - Two Controllers

Modal Assignment

Controller #1	Controller #2	Residual
1,2,4,5	3,6,7,8	None

Before Transformation

Time	Los-X	Los-Y
.5	.003649	.001223
1.0	.003929	.000133
1.5	.002813	-.000287
2.0	.001770	-.000055
2.5	.000959	.000193
3.0	.000207	.000442
3.5	-.000180	.000622
4.0	-.000215	.000609
4.5	-.000130	.000357
5.0	-.000027	-.000002
5.5	.000062	-.000316
6.0	.000113	-.000477
6.5	.000111	-.000443
7.0	.000063	-.000256
7.5	-.000006	-.000008
8.0	-.000079	.000202
8.5	-.000123	.000307
9.0	-.000127	.000287
9.5	-.000089	.000169
10.0	-.000024	.000013
10.5	.000044	-.000120
11.0	.000090	-.000189
11.5	.000100	-.000180
12.0	.000072	-.000110
12.5	.000022	-.000015
13.0	-.000030	.000070
13.5	-.000064	.000116
14.0	-.000070	.000115
14.5	-.000049	.000074
15.0	-.000014	.000015
15.5	.000022	-.000040
16.0	.000044	-.000073
16.5	.000047	-.000076
17.0	.000032	-.000052
17.5	.000008	-.000013
18.0	-.000014	.000023
18.5	-.000028	.000047
19.0	-.000030	.000050
19.5	-.000021	.000036
20.0	-.000006	.000011

After Transformation

Time	Los-X	Los-Y
.5	.003740	.001486
1.0	.003442	.000104
1.5	.001271	-.000839
2.0	-.000259	-.000930
2.5	-.000850	-.000701
3.0	-.001076	-.000204
3.5	-.000713	.000086
4.0	-.000331	.000597
4.5	.000185	.000416
5.0	.000219	.000433
5.5	.000290	-.000004
6.0	.000106	-.000079
6.5	.000055	-.000271
7.0	-.000024	-.000230
7.5	-.000060	-.000111
8.0	-.000002	-.000085
8.5	-.000060	.000132
9.0	.000044	.000011
9.5	-.000058	.000185
10.0	.000044	-.000029
10.5	-.000051	.000092
11.0	.000030	-.000075
11.5	-.000017	-.000001
12.0	.000013	-.000044
12.5	.000020	-.000042
13.0	-.000015	.000031
13.5	.000038	-.000056
14.0	-.000044	.000084
14.5	.000040	-.000066
15.0	-.000051	.000088
15.5	.000035	-.000064
16.0	-.000034	.000056
16.5	.000022	-.000040
17.0	-.000006	.000010
17.5	-.000001	.000003
18.0	.000019	-.000031
18.5	-.000026	.000045
19.0	.000034	-.000059
19.5	-.000040	.000068
20.0	.000038	-.000066

Table VIII

System Eigenvalue Analysis - Two Controller

Modal Assignment

Controller #1	Controller #2	Residual
1,2,4,5	3,6,7,8	9,10,11,12

Overall System Eigenvalues

Before Transformation	After Transformation
.07015 ± 13.96806 i	.07002 ± 13.96821 i
.05713 ± 10.34372 i	.05548 ± 10.34651 i
.06320 ± 10.94403 i	.06051 ± 10.94284 i
.05679 ± 8.99021 i	.05581 ± 8.94227 i
1.66583 ± 5.62161 i	.02837 ± 5.67583 i
1.36779 ± 5.20554 i	1.57647 ± 5.46629 i
1.61102 ± 5.66711 i	1.55483 ± 5.60263 i
.82780 ± 3.70680 i	1.03008 ± 5.52416 i
.16560 ± 1.24134 i	.82055 ± 3.65895 i
.61372 ± .61002 i	1.61266 ± 4.90964 i
1.33748 ± 5.23741 i	1.61259 ± 4.90962 i
1.61266 ± 4.90964 i	.17427 ± 1.20133 i
1.61259 ± 4.90962 i	.37831 ± 1.09691 i
1.27879 ± 2.99798 i	1.21939 ± 3.04764 i
1.09500 ± 3.92760 i	1.06961 ± 3.89587 i
.89774 ± .43082 i	.13284 ± 1.47589 i
.27864 ± 1.55548 i	.51475 ± 1.39833 i
1.45666 ± 3.31914 i	1.42381 ± 3.34208 i
.74323 ± 3.07171 i	.70898 ± 3.01768 i
1.03055 ± 2.52592 i	1.06814 ± 2.63964 i

Table VIII 4

Time Response - Two Controllers

Modal Assignment

	Controller #1 1,2,4,5	Controller #2 3,6,7,8	Residual 9,10,11,12	
	Before Transformation		After Transformation	
Time	Los-X	Los-Y	Los-X	Los-Y
.5	.003545	.001220	.003739	.001485
1.0	.003851	.000089	.003411	.000090
1.5	.002752	-.000341	.001232	-.000885
2.0	.001874	-.000067	-.000251	-.001027
2.5	.001189	.000223	-.000824	-.000845
3.0	.000535	.000543	-.001032	-.000335
3.5	.000276	.000807	-.000595	.000016
4.0	.000291	.000823	-.000161	.000583
4.5	.000281	.000542	.000310	.000451
5.0	.000228	.000129	.000235	.000491
5.5	.000181	-.000245	.000189	.000043
6.0	.000138	-.000452	-.000065	-.000056
6.5	.000092	-.000439	-.000115	-.000257
7.0	.000049	-.000252	-.000120	-.000212
7.5	.000005	.000003	-.000045	-.000085
8.0	-.000044	.000220	.000108	-.000054
8.5	-.000087	.000322	.000086	.000150
9.0	-.000110	.000290	.000153	-.000001
9.5	-.000099	.000160	-.000029	.000138
10.0	-.000050	-.000002	-.000009	-.000094
10.5	.000019	-.000132	-.000145	.000033
11.0	.000075	-.000193	-.000057	-.000101
11.5	.000093	-.000176	-.000054	.000017
12.0	.000083	-.000101	.000035	.000010
12.5	.000036	-.000005	.000081	.000025
13.0	-.000023	.000074	.000045	.000081
13.5	-.000065	.000115	.000068	-.000041
14.0	-.000076	.000110	-.000055	.000060
14.5	-.000056	.000069	-.000002	-.000113
15.0	-.000019	.000010	-.000098	.000038
15.5	.000023	-.000041	.000009	-.000093
16.0	.000049	-.000071	-.000027	.000055
16.5	.000051	-.000073	.000054	-.000015
17.0	.000034	-.000050	.000033	.000045
17.5	.000008	-.000013	.000024	.000031
18.0	-.000117	.000023	.000017	-.000020
18.5	-.000032	.000046	-.000050	.000037
19.0	-.000032	.000050	.000001	-.000078
19.5	-.000020	.000036	-.000064	.000048
20.0	-.000004	.000011	.000033	-.000077

mode shapes was accomplished. These are displayed in Appendix A. Using the definition of the dot product of two vectors:

$$\bar{A} \cdot \bar{B} = \left|\bar{A}\right| \left|\bar{B}\right| \cos \theta_{AB}$$

The angles between the modal amplitude vectors were determined. This was done to determine if any of the modes lied on lines of action such that they could either be simply separated or arranged to minimize control efforts required by associating similarly aligned modes. As a result of this investigation, it became evident that the modal amplitude vectors subdivided into two orthogonal vectors (Table 9).

With these orthogonal groupings, the system was run with controller one operating on modes 1, 4, 6, and 7; while controller number two drove modes 2, 3, 5, and 8. This grouping provided the best overall system response. The eigenvalues of this system is depicted in Table X while the associated time response is listed in Table Xa.

The unique quality of this system is that it is inherently decoupled, in that the associated feedback gain matrices of one system (K and G) are orthogonal to the other system parameter matrices (B and C). This results in the fact the off diagonal coupling terms B_2 G_1, K_2 C_1, K_1 C_2, B_1 G_2 are all equal to zero.

Table IX

Angular Relationships Between Modal Amplitude Vectors

Vector Dot Product

$\phi_1 \cdot \phi_3 = 0.0$

$\phi_1 \cdot \phi_6 = 0.0$

$\phi_1 \cdot \phi_7 = .06848$

$\phi_1 \cdot \phi_8 = 0.0$ $\qquad \theta_{17} = 33.23^{\circ}$

$\phi_2 \cdot \phi_3 = .03281$ $\qquad \theta_{23} = 64.33^{\circ}$

$\phi_2 \cdot \phi_6 = 0.0$

$\phi_2 \cdot \phi_7 = 0.0$

$\phi_2 \cdot \phi_8 = .08112$ $\qquad \theta_{28} = 50.12^{\circ}$

$\phi_4 \cdot \phi_3 = 0.0$

$\phi_4 \cdot \phi_6 = 0.0$

$\phi_4 \cdot \phi_7 = -.05228$

$\phi_4 \cdot \phi_8 = 0.0$ $\qquad \theta_{47} = 80.37^{\circ}$

$\phi_5 \cdot \phi_3 = .018290$ $\qquad \theta_{53} = 85.25^{\circ}$

$\phi_5 \cdot \phi_6 = 0.0$

$\phi_5 \cdot \phi_7 = 0.0$

$\phi_5 \cdot \phi_8 = .03614$ $\qquad \theta_{58} = 84.38^{\circ}$

This modal vector orthogonality while just a chance occurance, shows the importance of proper location of the sensors and actuators on the model. This system analysis to locate the sensors on the structure is a design tool which should not be taken lightly. The judicious location of sensors and actuators can reduce the system to a pair of uncoupled controllers requiring no system suppression; as a result, no degradation in the system response from the optimal gain values. By referring again to Table 7a, it is obvious that the time response of the system before suppression is superior to that after suppression.

Table X

System Eigenvalue Analysis - Two Controllers

Modal Assignments

Controller #1	Controller #2	Residual
1,4,6,7	2,3,5,8	9,10,11,12

Overall System Eigenvalues

Before Transformation	After Transformation
-.05702 ± 10.34654	-.07953 ± 10.20562
-.07011 ± 13.96812	-.06983 ± 13.96671
-.06244 ± 10.94332	-.04954 ± 10.93907
-.04388 ± 8.99752	-.05750 ± 8.96044
-1.51751 ± 5.50653	-1.61259 ± 4.90962
-1.61639 ± 5.41038	-1.22750 ± 4.97987
-.27191 ± 1.20414	-1.54666 ± 5.50147
-.43310 ± 1.09745	-1.58461 ± 5.40713
-.83306 ± 3.65462	-1.45632 ± 5.53310
-1.22213 ± 3.06310	-1.51763 ± 4.45319
-1.42714 ± 5.58897	-.54310 ± 3.63429
-1.58679 ± 5.40001	-1.06302 ± 3.29105
-.71819 ± 1.27304	-.36782 ± 1.13101
-.36226 ± 1.51909	-.26965 ± 1.18851
-1.06186 ± 2.50015	-.43999 ± 3.85085
-.73434 ± 3.05232	-1.25745 ± 3.65065
-1.43137 ± 3.35652	-.62836 ± 1.31448
-1.08042 ± 3.88162	-.34492 ± 1.51727
-1.61259 ± 4.90962	-.73034 ± 3.04703
-1.61266 ± 4.90964	-1.09035 ± 2.51427

Table Xa

Time Response - Two Controllers

Modal Assignments

Controller #1	Controller #2	Residual
1,4,6,7	2,3,5,8	9,10,11,12

Before Transformation			After Transformation		
Time	Los-X	Los-Y	Time	Los-X	Los-Y
.5	.003596	.001203	.5	.003791	.001292
1.0	.003360	-.000289	1.0	.003580	.000704
1.5	.001491	-.001148	1.5	.002826	-.000367
2.0	.000163	-.001043	2.0	-.000043	-.001124
2.5	-.000598	-.000703	2.5	-.000766	-.000740
3.0	-.000939	-.000256	3.0	-.001248	-.000382
3.5	-.000707	.000278	3.5	-.001542	-.000154
4.0	-.000198	.000628	4.0	-.000518	.000484
4.5	.000143	.000625	4.5	.000415	.000808
5.0	.000216	.000376	5.0	.000402	.000489
5.5	.000143	.000054	5.5	.000324	.000144
6.0	.000035	.000200	6.0	.000277	-.000068
6.5	-.000039	-.000304	6.5	-.000048	-.000336
7.0	-.000050	-.000263	7.0	-.000219	-.000384
7.5	-.000013	-.000136	7.5	-.000093	-.000201
8.0	.000031	.000004	8.0	-.000012	-.000029
8.5	.000049	.000100	8.5	.000017	.000084
9.0	.000030	.000126	9.0	.000085	.000167
9.5	-.000008	.000095	9.5	.000067	.000150
10.0	-.000037	.000040	10.0	-.000021	.000061
10.5	-.000041	-.000010	10.5	-.000051	-.000006
11.0	-.000026	-.000041	11.0	-.000042	-.000044
11.5	-.000002	-.000046	11.5	-.000036	-.000064
12.0	.000021	-.000032	12.0	-.000002	-.000048
12.5	.000030	-.000011	12.5	.000037	-.000013
13.0	.000021	.000005	13.0	.000038	.000007
13.5	.000006	.000013	13.5	.000020	.000016
14.0	-.000007	.000016	14.0	.000005	.000019
14.5	-.000016	.000011	14.5	-.000013	.000012
15.0	-.000016	.000005	15.0	-.000024	.000002
15.5	-.000008	.000000	15.5	-.000017	-.000002
16.0	.000002	-.000003	16.0	-.000005	-.000003
16.5	.000006	-.000005	16.5	.000004	-.000004
17.0	.000008	-.000005	17.0	.000010	-.000003
17.5	.000007	-.000003	17.5	.000011	-.000001
18.0	.000002	-.000001	18.0	.000005	-.000001
18.5	-.000003	.000000	18.5	-.000001	-.000001
19.0	-.000004	.000002	19.0	-.000004	.000000
19.5	-.000003	.000003	19.5	-.000005	.000001
20.0	-.000002	.000002	20.0	-.000004	.000001

Conclusions

During this study it was determined that system response can be greatly reduced through the implementation of an additional controller. As became evident in the suppression portion of the research, the designer can greatly reduce the computational requirements through the use of structural symmetry and sensor locations. By assigning modes to be controlled according to orthogonal grouping of their modal ampitudes associated to each sensor location, the system will be inherently decoupled as earlier explained.

In all of the test cases run, the residual modes were not adversely affected by any of the control or transformation techniques applied to the overall system. As a result, including only the lower freqency modes as controlled modes, has proved valid for the modeled system.

The capability to control the system may be increased through additional sensors, but it must be noted that the system will not be able to suppress more modes than sensors as was noted in the transformation section.

Recommendations

The primary thrust of this investigation was toward the evaluation of a system which implemented two decentralized controllers. The results presented indicate the mathematical advantages of applying this technique to the model chosen. The importance of evaluating the entire modal analysis became evident through the analysis of the modal amplitude vectors. This single area has presented itself as a key to real world application of decentralized controllers. The importance of the location of the sensors and actuators that are used to control the structural motion is an important design tool in achieving desired system response.

The next logical step in the study of this control problem would be the experimental evaluation of the techniques applied in this study to determine the feasibility of the implementation of the system described. This would include the evaluation and determination of computing capabilities required to achieve the results which have been put forth in this paper.

Bibliography

1. Balas, M. J., "Active Control of Flexible Systems", AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, June 14, 1977.

2. Sesak, J. R., "Control of Large Space Structures Via Singular Perturbation Optimal Control", AIAA Conference on Large Space Platforms: Future Needs and Capabilities, Los Angeles, California, September 27-29, 1978.

3. Coradetti, T., "Orthogonal Subspace Reduction of Optimal Regulator Order", General Dynamics/Convair Division, San Diego, California.

4. Strang, G., Linear Algebra and Its Applications, New York: Academic Press, 1976.

5. Calico, R. A., Jr., and Janiszewski, A. M., "Control of a Flexible Satellite Via Elimination of Control Spillover," Proceeds Third VPI/AIAA Symposium on Large Space Structures, Blacksburg, Virginia, June 1981.

6. Kleinman, D. L., A Description of Computer Programs for Use in Linear Systems Studies. The University of Connecticut School of Engineering TR-77-2, Storrs, Connecticut, July 1977.

7. D'Azzo, J. J. and Houpis, C. H., Linear Control System Analysis and Design: Conventional and Modern, New York: McGraw-Hill Book Company, 1975.

Appendix A

NASTRAN Analysis

Frequency and Mode Shapes

Nominal Case

$$\omega_1 = 1.370 \qquad \phi_1 = \begin{bmatrix} -2.471\text{E-}01 \\ 4.279\text{E-}02 \\ 1.451\text{E-}06 \\ -1.963\text{E-}02 \\ 3.398\text{E-}02 \\ -7.213\text{E-}02 \\ -3.607\text{E-}02 \\ 4.347\text{E-}02 \\ 4.397\text{E-}02 \\ -1.962\text{E-}02 \\ 5.296\text{E-}02 \\ 4.397\text{E-}02 \end{bmatrix}$$

$$\omega_2 = 1.467 \qquad \phi_2 = \begin{bmatrix} 3.999\text{E-}01 \\ 2.309\text{E-}01 \\ -1.489\text{E-}01 \\ 8.329\text{E-}02 \\ 4.808\text{E-}02 \\ 6.813\text{E-}02 \\ 7.090\text{E-}02 \\ 2.253\text{E-}02 \\ -4.721\text{E-}02 \\ 5.451\text{E-}02 \\ 4.936\text{E-}02 \\ -4.722\text{E-}02 \end{bmatrix}$$

$$\omega_3 = 2.965 \qquad \phi_3 = \begin{bmatrix} 6.368\text{E-}02 \\ 3.678\text{E-}02 \\ 4.000\text{E-}01 \\ 1.984\text{E-}01 \\ 1.145\text{E-}01 \\ 2.010\text{E-}01 \\ 1.548\text{E-}01 \\ 6.804\text{E-}02 \\ 9.782\text{E-}02 \\ 1.363\text{E-}01 \\ 1.000\text{E-}01 \\ 9.784\text{E-}02 \end{bmatrix}$$

$$\omega_4 = 3.502 \qquad \phi_4 = \begin{bmatrix} 2.746\text{E-}02 \\ -4.758\text{E-}02 \\ -2.249\text{E-}05 \\ -1.718\text{E-}01 \\ 2.977\text{E-}01 \\ -6.817\text{E-}05 \\ -2.512\text{E-}01 \\ 3.436\text{E-}01 \\ -8.190\text{E-}02 \\ -1.718\text{E-}01 \\ 3.894\text{E-}01 \\ 8.192\text{E-}02 \end{bmatrix}$$

$$\omega_5 = 3.848 \qquad \phi_5 = \begin{bmatrix} -8.783\text{E-}02 \\ -5.070\text{E-}02 \\ -1.299\text{E-}01 \\ 3.095\text{E-}01 \\ 1.786\text{E-}01 \\ -3.514\text{E-}01 \\ 2.866\text{E-}01 \\ 1.224\text{E-}01 \\ 1.139\text{E-}02 \\ 2.494\text{E-}01 \\ 1.868\text{E-}01 \\ 1.140\text{E-}02 \end{bmatrix}$$

$$\omega_6 = 5.150 \qquad \phi_6 = \begin{bmatrix} 1.353\text{E-}05 \\ 1.218\text{E-}11 \\ 3.402\text{E-}11 \\ -2.041\text{E-}01 \\ 3.535\text{E-}01 \\ -6.057\text{E-}06 \\ -2.041\text{E-}01 \\ -3.535\text{E-}01 \\ 1.086\text{E-}04 \\ 4.082\text{E-}01 \\ 6.802\text{E-}10 \\ 5.065\text{E-}10 \end{bmatrix}$$

$$\omega_7 = 5.676 \qquad \phi_7 = \begin{bmatrix} -2.661\text{E-}02 \\ 4.607\text{E-}02 \\ 3.302\text{E-}05 \\ 3.374\text{E-}02 \\ -5.844\text{E-}02 \\ 3.231\text{E-}05 \\ 2.733\text{E-}02 \\ -5.481\text{E-}02 \\ -4.313\text{E-}01 \\ 3.382\text{E-}02 \\ -5.108\text{E-}02 \\ 4.908\text{E-}01 \end{bmatrix}$$

$$\omega_8 = 5.680 \qquad \phi_8 = \begin{bmatrix} -2.994\text{E-}02 \\ -1.731\text{E-}02 \\ 8.784\text{E-}02 \\ 4.071\text{E-}02 \\ 2.360\text{E-}02 \\ 3.554\text{E-}02 \\ 2.742\text{E-}02 \\ 2.798\text{E-}02 \\ -4.875\text{E-}01 \\ 3.799\text{E-}02 \\ 9.810\text{E-}03 \\ -4.879\text{E-}01 \end{bmatrix}$$

$$\omega_9 = 8.940 \qquad \phi_9 = \begin{bmatrix} 9.907\text{E-}02 \\ 5.720\text{E-}02 \\ 1.729\text{E-}01 \\ 1.076\text{E-}01 \\ 6.213\text{E-}02 \\ -4.953\text{E-}01 \\ -1.679\text{E-}01 \\ -2.198\text{E-}01 \\ -1.110\text{E-}02 \\ -2.743\text{E-}01 \\ -3.554\text{E-}02 \\ -1.109\text{E-}02 \end{bmatrix}$$

$$\omega_{10} = 10.303 \qquad \phi_{10} = \begin{bmatrix} -3.390\text{E-}03 \\ 5.850\text{E-}03 \\ -1.605\text{E-}05 \\ -2.286\text{E-}01 \\ 3.960\text{E-}01 \\ 4.964\text{E-}05 \\ 3.783\text{E-}01 \\ 4.554\text{E-}02 \\ -1.471\text{E-}02 \\ -2.286\text{E-}01 \\ -3.049\text{E-}01 \\ 1.472\text{E-}02 \end{bmatrix}$$

$$\omega_{11} = 10.923 \qquad \phi_{11} = \begin{bmatrix} 6.370\text{E-}02 \\ 3.678\text{E-}02 \\ 3.588\text{E-}02 \\ -2.401\text{E-}01 \\ -1.385\text{E-}01 \\ -2.605\text{E-}01 \\ -8.626\text{E-}02 \\ 3.944\text{E-}01 \\ 6.970\text{E-}03 \\ 2.984\text{E-}01 \\ -2.719\text{E-}01 \\ 6.971\text{E-}03 \end{bmatrix}$$

$$\omega_{12} = 13.966 \qquad \phi_{12} = \begin{bmatrix} 3.206\text{E-}02 \\ 1.851\text{E-}02 \\ 6.438\text{E-}02 \\ -4.026\text{E-}01 \\ -2.324\text{E-}01 \\ -1.305\text{E-}01 \\ 3.204\text{E-}01 \\ -1.587\text{E-}01 \\ -9.278\text{E-}03 \\ 2.272\text{E-}02 \\ 3.568\text{E-}01 \\ -9.281\text{E-}03 \end{bmatrix}$$

Frequency and Mode Shapes

Perturbed Case

$\omega_1 = 1.342$

$$\phi_1 = \begin{bmatrix} -3.444E-01 \\ 5.964E-01 \\ 2.330E-06 \\ -3.107E-02 \\ 5.379E-02 \\ -1.111E-05 \\ -5.079E-02 \\ 6.518E-02 \\ 6.380E-02 \\ -3.101E-02 \\ 7.656E-02 \\ -6.379E-02 \end{bmatrix}$$

$\omega_2 = 1.665$

$$\phi_2 = \begin{bmatrix} 5.429E-01 \\ 3.135E-01 \\ -1.990E-01 \\ 1.263E-01 \\ 7.292E-02 \\ 9.756E-02 \\ 1.098E-01 \\ 4.162E-02 \\ -6.728E-02 \\ 9.092E-02 \\ 7.425E-02 \\ -6.728E-02 \end{bmatrix}$$

$\omega_3 = 2.891$

$$\phi_3 = \begin{bmatrix} -4.421E-02 \\ -2.844E-02 \\ 3.788E-01 \\ 3.125E-01 \\ 1.805E-01 \\ 6.519E-02 \\ 2.727E-01 \\ 1.274E-01 \\ 1.371E-01 \\ 2.466E-01 \\ 1.726E-01 \\ 1.372E-01 \end{bmatrix}$$

$\omega_4 = 2.957$

$$\phi_4 = \begin{bmatrix} 5.726E-02 \\ -9.915E-02 \\ -1.466E-04 \\ -1.760E-01 \\ 3.046E-01 \\ -7.205E-05 \\ -2.388E-01 \\ 3.409E-01 \\ -9.157E-02 \\ -1.759E-01 \\ 3.771E-01 \\ 9.149E-02 \end{bmatrix}$$

$\omega_5 = 3.398$

$$\phi_5 = \begin{bmatrix} -1.369E-01 \\ -7.906E-02 \\ -3.441E-01 \\ 1.621E-01 \\ 9.356E-02 \\ -4.969E-02 \\ 1.620E-01 \\ 7.309E-02 \\ -7.571E-02 \\ 1.444E-01 \\ 1.037E-01 \\ -7.570E-02 \end{bmatrix}$$

$\omega_6 = 4.205$

$$\phi_6 = \begin{bmatrix} 2.706E-05 \\ 2.487E-11 \\ 6.986E-11 \\ -2.041E-01 \\ 3.535E-01 \\ 5.160E-06 \\ -2.041E-01 \\ -3.535E-01 \\ 1.003E-04 \\ 4.082E-10 \\ 7.861E-10 \\ 6.086E-10 \end{bmatrix}$$

$\omega_7 = 4.662$

$$\phi_7 = \begin{bmatrix} 5.571E-02 \\ -9.647E-02 \\ -2.248E-05 \\ -3.440E-02 \\ 5.960E-02 \\ -2.905E-05 \\ -2.882E-02 \\ 5.644E-02 \\ 4.873E-01 \\ -3.447E-02 \\ 5.318E-02 \\ -4.872E-01 \end{bmatrix}$$

$\omega_8 = 4.755$

$$\phi_8 = \begin{bmatrix} -7.584E-02 \\ -4.380E-02 \\ 1.837E-01 \\ 4.701E-02 \\ 2.722E-02 \\ 9.781E-02 \\ 3.671E-02 \\ 3.245E-02 \\ -4.695E-01 \\ 4.655E-02 \\ 1.565E-02 \\ -4.698E-01 \end{bmatrix}$$

$\omega_9 = 8.539$

$$\phi_9 = \begin{bmatrix} 1.445E-01 \\ 8.347E-02 \\ 2.702E-01 \\ 2.125E-01 \\ 1.228E-01 \\ -3.266E-01 \\ -1.414E-01 \\ -3.096E-01 \\ -1.504E-02 \\ -3.389E-01 \\ -3.228E-02 \\ -1.503E-02 \end{bmatrix}$$

$\omega_{10} = 9.251$

$$\phi_{10} = \begin{bmatrix} -5.777E-03 \\ 9.965E-03 \\ -3.372E-05 \\ -2.242E-01 \\ 3.883E-01 \\ 4.517E-05 \\ 3.846E-01 \\ 3.681E-02 \\ -1.184F-02 \\ -2.241E-01 \\ -2.147E-01 \\ 1.185E-02 \end{bmatrix}$$

$\omega_{11} = 10.285$

$$\phi_{11} = \begin{bmatrix} 1.594E-01 \\ 9.205E-02 \\ 2.580E-01 \\ -1.516E-01 \\ -8.758E-02 \\ -3.117E-01 \\ -1.619E-01 \\ 3.311E-01 \\ 9.133E-04 \\ 2.057E-01 \\ -3.058E-01 \\ 9.153E-04 \end{bmatrix}$$

$\omega_{12} = 12.905$

$$\phi_{12} = \begin{bmatrix} 8.369E-02 \\ 4.833E-02 \\ 1.587E-01 \\ -4.059E-01 \\ -2.343E-01 \\ -1.611E-01 \\ 2.996E-01 \\ -1.419E-01 \\ -8.200E-03 \\ 2.687E-02 \\ 3.304E-01 \\ -8.203E-03 \end{bmatrix}$$

ϕ^T D Matrix

Mode	Actuators 1	2	3	4	5	6
1	0.044	-0.044	-0.067	-0.023	0.023	0.067
2	0.069	-0.069	-0.017	0.112	0.112	-0.017
3	-0.046	-0.046	-0.271	0.077	0.077	-0.271
4	0.248	-0.249	-0.060	0.189	-0.189	0.060
5	0.351	0.351	-0.049	0.156	0.156	-0.049
6	0.289	-0.289	0.289	-0.289	0.289	-0.289
7	0.049	-0.049	-0.369	-0.320	0.320	0.369
8	-0.069	-0.069	0.299	0.365	0.365	0.299
9	0.231	0.231	-0.250	-0.229	-0.229	0.250
10	0.317	-0.317	-0.150	0.167	-0.167	-0.150
11	0.220	0.220	-0.146	0.145	0.145	-0.146
12	0.114	0.114	-0.013	0.025	0.0248	-0.013

Appendix B

Main Program Listing

```
      PROGRAM THESIS
      REAL MODE(2,12),XI(40),INIT(4,12),XD(40)
      REAL FSBT(12,12),RIC(12,12),ABS2(12,12),AKO2(12,12),GAIN2(12,12)
      REAL DT,EAT1(40,40),EAT(40,40),EAT2(40,40)
      REAL ACT(12,12),BT(12,12),ZETA,W(12)
      REAL KT2(12,12),CST(12,12),CSTR(12,12),TRT(12,12)
      REAL FT(12,12),TCD(12,12),CTT(12,12),RT1(12,12)
      REAL AC(12,12),D(12),PHIS(12,12),DD(12,12)
      REAL CAT(12,12),ABG(12,12),SAT(12,12),BRG(12,12),KOR(12,12)
      REAL MAJM(40,40),GAIN(12,12),BCG(12,12),AKC(12,12)
      REAL KCS(12,12),BSG(12,12),CUB(12,12),CT(12,12)
      REAL V(12,12),SING(12),IR(12,12)
      REAL CTCD(12,12),POB(12,12),ACG(12,12),KT1(12,12)
       REAL XTR(12,12),STOR(12,12),TOL,TEN(12,12),AA,BB,DA(12,12)
      REAL FST(12,12),T1(12,12),TT(12,12),R(12,12),RI(12,12)
      REAL FC(12,12),PHI(12,12)
      INTEGER N,NC,NS,NR,IC(12),IC2,NR2,NS2,MM,L,P,M,SKIP,DEC,E
      INTEGER I,IER,J,ZZ,N2,Q,TAPE, IS(12),NACT,IR(12)
      COMPLEX W1(12),Z(4,4)
      REAL KOB(12,12),KOC(12,12),WORK(40,40)
      COMMON/MAIN1/NDIM,NDIM1,TEN
      COMMON/MAIN4/NDA,NDA1,WORK
      COMMON/MAIN2/STOR
       COMMON/MAIN3/XTR
      COMMON/INOUT/TAPE
      COMMON/NUM/IC,IS,IR,NC,NS,NR
      COMMON/SAVE/T(100),TS(100)
      NDIM=12
      NDIM1=13
      NDA=40
      NDA1=33
      Q=0
      TAPE=9
      PRINT*,'ENTER NC,NS,NR,NACT,NSEN,ZETA> '
      READ*,NC,NS,NR,NACT,NSEN,ZETA
       PRINT*,' '
      PRINT*,' ENTER THE ',NACT,' ELEMENTS FOR EACH PHIA'
      N=NC+NS+NR
      DO 5 I=1,N
      PRINT*,'ENTER PHIA ',I,'>'
       READ(5,*)(PHI (I,J),J=1,NACT)
5     CONTINUE
       PRINT*,' '
      PRINT*,' ENTER THE ',NSEN,' ELEMENTS FOR EACH PHIS'
      DO 6 I=1,N
      PRINT*,'ENTER PHIS ',I,'>'
       READ(5,*)(PHIS(I,J),J=1,NSEN)
6      CONTINUE
      DO 4 I=1,N
      PRINT*,' ENTER OMEGA ',I,' >'
      READ(5,*)W(I)
      D(I)= -2*ZETA*W(I)
4     CONTINUE
C
C
C     INITIAL CONDITIONS ARE READ IN BY ROW
C     ROW 1 IS X , ROW 2 IS X DOT , ROW 3 IS E, ROW 4 IS E DOT
```

```
C
C
      DO 19 I=1,2
19    READ(F,*)(MODE(I,J),J=1,N)
      DO 21 I=1,6
21    READ(8,*)(INIT(I,J),J=1,N)
299   CONTINUE
      DEC=0
      PRINT*,'   IF TIS IS A DECOUPLED RUN ENTER 1 ELSE ENTER 0 >'
      READ*,DEC
      PRINT*,' DECOUPLE = ',DEC
      IF(0.EQ.2) THEN
      PRINT*,'   ENTER NC,NS,NR > '
      READ*,NC,NS,NR
      ENDIF
      PRINT*,' ENTER THE ',NC,' CONTROLLED MODES > '
      READ*,(IC(I),I=1,NC)
      PRINT*,'         ',(IC(I),I=1,NC)
      PRINT*,'ENTER THE ',NS,' SUPRESSED MODES > '
      READ*,(IS(I),I=1,NS)
      PRINT*,'         ',(IS(I),I=1,NS)
      IF (NR.NE.0) THEN
      PRINT*,' ENTER THE ',NR,' RESIDUAL MODES > '
      READ*,(IR(I),I=1,NR)
      PRINT*,'         ',(IR(I),I=1,NR)
      ENDIF
      NC2=2*NC
      NR2=2*NR
      NS2=NS*2
      N2=2*N
      CALL FORMX0(X0,INIT)
      IF(DEC.EQ.1)CALL FORMX1(X0,INIT)
      M=2*NC2+NS2+NR2
      IF(DEC.EQ.1)M=2*NC2+2*NS2+NR2
      PRINT*,'   INITIAL CONDITIONS'
      CALL PRNT(X0,M,1)
      PRINT*,'   TO PRINT ALL OF THE MATRICIES ENTER 1, ELSE ENTER  >'
      READ*,O
      IF (O.EQ.1) THEN
      PRINT*,' THE A CONTROL MATRIX IS'
      CALL FORMA(AC,D,W,NC,NC2,IC)
      CALL PRNT(AC,NC2,NC2)
      PRINT*,' THE B CONTROLLED MATRIX IS '
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      CALL PRNT(BC,NC2,NACT)
       PRINT*,' THE C CONTROLLED MATRIX IS'
       CALL FORMC(CC,PHIS,NC,NC2,NSEN,IC)
       CALL PRNT(CC,NSEN,NC2)
      PRINT*,' THE A SUPRESSED MATRIX IS'
      CALL FORMA(AC,D,W,NS,NS2,IS)
      CALL PRNT(AC,NS2,NS2)
      PRINT*,' THE B SUPRESSED MATRIX IS'
      CALL FORMB(BC,PHI,NS,NS2,NACT,IS)
      CALL PRNT(BC,NS2,NACT)
       PRINT*,' THE C SUPPRESSED MATRIX IS'
      CALL FORMC(CC,PHIS,NS,NS2,NSEN,IS)
       CALL PRNT(CC,NSEN,NS2)
```

```
      PRINT*,' THE A RESIDUAL MATRIX IS'
      CALL FORMA(AC,D,W,NC,NC2,IR)
       CALL PRNT(AC,NR2,NR2)
      PRINT*, ' THE B RESIDUAL MATRIX IS'
      CALL FORMB(BC,PHI,NR,NR2,NACT,IR)
      CALL PRNT(BC,NR2,NACT)
       PRINT*,' THE C RESIDUAL MATRIX IS'
      CALL FORMC(CC,PHIS,NR,NR2,NSEN,IR)
       CALL PRNT(CC,NSEN,NR2)
      Q=0
      ENDIF
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      CALL TPR(BT,BC,NC2,NACT,1,2)
      CALL MMUL(BC,BT,NC2,NACT,NC2,SAT)
      CALL FORMC(CC,PHIS,NC,NC2,NSEN,IC)
      CALL TPR(CT,CC,NSEN,NC2,1,2)
      CALL MMUL(CT,CC,NC2,NSEN,NC2,CTCC)
121   CONTINUE
      ZZ=0
      PRINT*,' ENTER THE DIAGONAL TERM FOR THE WEIGHTING MATRIX Q >'
      READ*,AA
      PRINT*,AA
      PRINT*,' ENTER THE OBSERVER WEIGHTING DIAGONAL TERM >'
      READ*,BB
      PRINT*,BB
      DO 100 I=1,NC2
      DO 150 J=1,NC2
      IF(I.EQ.J) THEN
      QA(I,J)=AA
      QOB(I,J)=BB
      ELSE
      QA(I,J)=
      QOB(I,J)=
      ENDIF
150   CONTINUE
100   CONTINUE
      IER=
      TOL=. 1
      CALL FORMA(AC,D,W,NC,NC2,IC)
      CALL MRIC(NC2,AC,SAT,QA,CAT,RES,TOL,IER)
      IF(ZZ.EQ. ) THEN
      PRINT*,' THE RICATTI SOLUTION OF AC + BCG'
      PRINT*,'  IER= ',IER
      CALL PRNT(CAT,NC2,NC2)
      ENDIF
      IER=
      TOL=. 1
      CALL TPR(ACT,AC,NC2,NC2,1,2)
      CALL MRIC(NC2,ACT,CTCC,QOB,POB,ABS,TOL,IER)
      CALL MMUL(CC,POB,NSEN,NC2,NC2,KT1)
      IF(ZZ.EQ.1) THEN
      CALL MMUL(KT1,TRT,P,P,NSEN,STOR)
      CALL MMUL(STOR,KT1,P,NSEN,NC2,KT2)
      CALL MMUL(TI,KT2,NSEN,P,NC2,KT1)
      ENDIF
      CALL TPR(KOS,KT1,NSEN,NC2,1,2)
      PRINT*,'   THE K GAIN MATRIX   '
```

```
      CALL FENT(KCB,NC2,NSEN)
      CALL MMUL(KCB,CC,NC2,NSEN,NC2,KCC)
      CALL FORMC(CC,PHIS,NS,NS2,NSEN,IS)
      CALL MMUL(KCB,CC,NC2,NSEN,NS2,KCS)
      CALL FORMC(CC,PHIS,NR,NR2,NSEN,IR)
       CALL MMUL(KCB,CC,NC2,NSEN,NR2,KCR)
      DO 87 I=1,NC2
      DO 87 J=1,NC2
87    AKC(I,J) = AC(I,J) - KCC(I,J)
      MM=NC2+NC2+NS2+NR2
      IF(DEC.EQ.1)MM=2*NC2+2*NS2+NR2
      DO 91 I=1,MM
      DO 91 J=1,MM
91    MAJM(I,J)= ..
      DO 92 I=1,NC2
      DO 92 J=1,NC2
92    MAJM(I,J)=ABG(I,J)
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      CALL TFR(BT,BC,NC2,NACT,1,2)
      CALL MMUL(BT,CAT,NACT,NC2,NC2,GAIN)
      DO 77 I=1,NACT
      DO 77 J=1,NC2
77    GAIN(I,J)=-GAIN(I,J)
      CALL MMUL(BC, GAIN,NC2,NACT,NC2,BCG)
      CALL FORMB(BC,PHI,NS,NS2,NACT,IS)
      CALL MMUL(BC,GAIN,NS2,NACT,NC2,BSG)
      CALL FORMB(BC,PHI,NR,NR2,NACT,IR)
      CALL MMUL(BC,GAIN,NR2,NACT,NC2,BRG)
      L=2*NC2
      DO 93 I=1,NC2
      DO 93 J=1,NC2
93    MAJM(I,(J+NC2))= BCG(I,J)
      DO 94  I=1,NC2
      DO 94  J=1,NC2
94    MAJM((I+NC2),(J+NC2)) = AKC(I,J)
      DO 95 I=1,NC2
      DO 95 J=1,NS2
95    MAJM((I+NC2),(J+L)) =KCS(I,J)
      DO 96 I=1,NS2
      DO 96 J=1,NC2
      MAJM((L+I), J      )=BSG(I,J)
96    MAJM((L+I),(J+NC2))=BSG(I,J)
      CALL FORMA( AC,D,W,NS,NS2,IS)
      DO 97 I=1,NS2
      DO 97 J=1,NS2
97    MAJM((I+L),(J+L))=AC(I,J)
      M=L+NS2
      CALL FORMA(AC,D,W,NR,NR2,IR)
      DO 3  I=1,NR2
      DO 3  J=1,NR2
3     MAJM((I+M),(J+M)) =  AC(I,J)
      DO 31 I=1,NR2
      DO 31 J=1,NC2
      MAJM((I+M),J       ) =BRG(I,J)
      MAJM((I+M),(J+NC2)) =BRG(I,J)
31    MAJM((J+NC2),(I+M)) = KCR(J,I)
      IF(DEC.EQ.1) THEN
```

```
      DO 40 I=1,NR2
      DO 40  J=1,NC2
      MAJM((I+M),(J+NC2)) =0.0
      MAJM((I+M),J) =   0.0
40    MAJM((J+NC2),(I+M)) = 0.0
      CALL FORMA(AC,E,W,NS,NS2,IS)
      CALL FORMB(BC,PHI,NS,NS2,NACT,IS)
      CALL TFR(BT,BC,NS2,NACT,1,2)
      IF(ZZ.EQ.0) CALL MMUL(BC,BT,NS2,NACT,NS2,BSBT)
      IER=0
      TOL=.001
      CALL MRIC(NS2,AC,BSBT,QA,KIC,ABG2,TOL,IER)
      CALL TFR(ACT,AC,NS2,NS2,1,2)
      CALL FORMC(CC,PHIS,NS,NS2,NSEN,IS)
      CALL TFR(CT,CC,NSEN,NS2,1,2)
      CALL MMUL(CT,CC,NS2,NSEN,NS2,CTCC)
      CALL MRIC(NS2,ACT,CTCC,QOB,PCB,ACG,TOL,IER)
      CALL TFR(AKC2,ACG,NS2,NS2,1,2)
      M=2*NC2
      DO 41 I=1,NS2
      DO 41 J=1,NS2
      MAJM((M+I),(M+J)) = ABG2(I,J)
41    MAJM((M+NS2+I),(M+NS2+J)) = AKC2(I,J)
      CALL MMUL(CC,PCB,NSEN,NS2,NS2,KT1)
      CALL TFR(KOB,KT1,NSEN,NS2,1,2)
C    KOB IS NOW THE K GAIN MATRIX FOR SYSTEM 2
      M=M+NS2
      CALL FORMC(CC,PHIS,NC,NC2,NSEN,IC)
      CALL MMUL(KOB,CC,NS2,NSEN,NC2,KCC)
      DO 42 I=1,NS2
      DO 42 J=1,NC2
42    MAJM((M+I),J) = KCC(I,J)
      CALL MMUL(BT,KIC,NACT,NS2,NS2,GAIN2)
      DO 73 I=1,NACT
      DO 73 J=1,NS2
73    GAIN2(I,J)=-GAIN2(I,J)
      IF(ZZ.EQ.1)THEN
      CALL MMUL(TT,GAIN2,E,NACT,NS2,STOR)
      CALL MMUL(R1,STOR,E,E,NS2,TEN)
      CALL MMUL(T1,TEN,NACT,E,NS2,GAIN2)
      ENDIF
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      CALL MMUL(BC,GAIN2,NC2,NACT,NS2,BCG)
      M=NC2*2
      DO 43 I=1,NC2
      DO 43 J=1,NS2
      MAJM(I,(M+J)) = BCG(I,J)
43    MAJM(I,(M+NS2+J)) = BCG(I,J)
      CALL FORMB(BC,PHI,NS,NS2,NACT,IS)
      CALL MMUL(BC,GAIN2,NS2,NACT,NS2,BSG)
      DO 44 I=1,NS2
      DO 44 J=1,NS2
44    MAJM((M+I),(M+NS2+J)) = BSG(I,J)
      M=2*NC2+2*NS2
      DO 45 I=1,NR2
      DO 45 J=1,NC2
      MAJM((M+I),J) = BCG(I,J)
```

```
      MAJM((M+I),(J+NC2)) = BRG(I,J)
45    MAJM((NC2+J),(M+I)) = KOR(J,I)
      CALL FORMB(BC,PHI,NR,NR2,NACT,IR)
      CALL MMUL(BC,GAIN2,NR2,NACT,NS2,BRG)
      CALL FORMC(CC,PHIS,NR,NR2,NSEN,IR)
      CALL MMUL(KOR,CC,NS2,NSEN,NR2,KOR)
      DO 46 I=1,NR2
      DO 46 J=1,NS2
      MAJM((M+I),(J+2*NC2)) = BRG(I,J)
      MAJM((M+I),(J+2*NC2+NS2)) = ERG(I,J)
46    MAJM((2*NC2+NS2+J),(M+I)) = KOR(J,I)
      CALL FORMA(AC,C,W,NR,NR2,IF)
      DO 47 I=1,NR2
      DO 47 J=1,NR2
47    MAJM((M+I),(M+J)) = AC(I,J)
      ENDIF
      IF(DEC.EQ.1)MM=  2*NC2+2*NS2+NR2
C
C
C          FORMS E TO THE AT
C
C
      IF (ZZ.EQ.1) THEN
      PRINT*,'   THE SUPRESSED ANALYSIS IS CALCULATED'
      ENDIF
      PRINT*,'   FOR THE TIME RESPONSE AND EIGENVALUE ANALYSIS ENTER 1'
      PRINT*,'   FOR ONLY THE EIGENVALUE ANALYSIS ENTER  2 >'
      READ*,SKIP
      IF (SKIP.EQ.2) THEN
      GOTO 211
      ENDIF
       DT= 0.01
      TOL=.001
      DO 98 I=1,MM
      DO 98 J=1,MM
      EAT1(I,J)= 0.
      EAT1(I,I)=1.
      WORK(I,J)=MAJM(I,J)*DT
98    EAT(I,J)=WORK(I,J)
      M=1
      LL=1
112   CONTINUE
      DO 111 I=1,MM
      DO 111 J=1,MM
111   EAT2(I,J)=EAT1(I,J)+WORK(I,J)/LL
      M=M+1
      LL=LL*M
      DO 111 I=1,MM
      DO 111 J=1,MM
      IF(ABS(EAT2(I,J)-EAT1(I,J)).GT.TOL) THEN
      DO 113 L=1,MM
      DO 113 K=1,MM
      EAT1(L,K)=EAT2(L,K)
113   EAT2(L,K) = WORK(L,K)
      CALL VMULFF(EAT2,EAT,MM,MM,MM,MM,MM,WORK,MM,IER)
      GOTO 112
      ENDIF
```

```
111   CONTINUE
C
C
C     THE SOLUTION TO E TO THE AT IS IN EAT2
C     THIS BLOCK DETERMINES THE LOS AND PRINTS THIS VALUE EVERY 2. SEC
C
C
      IF(DEC.EQ.0) THEN
      CALL FORMXO(XO,INIT)
      ELSE
      CALL FORMX1(XO,INIT)
      ENDIF
      CALL TIME(EAT2,MM,DT,X1,XO,MODE,EAT,WORK,DEC)
210   CONTINUE
C
C
C      EIGEN VALUE ANALYSIS SECTION
      CALL EIGRF(MAJM,MM,40,0,Z,TEN,NDIM,WORK,IER)
C
C
      PRINT*,'   OVERALL SYSTEM EIGEN VALUES'
      PRINT*,'   IER = ',IER
      DO 65 I=1,MM
65    PRINT*,'          ',Z(I)
      PRINT*,' '
      CALL EIGRF(ABG,NO2,NDIM,0,W1,TEN,NDIM,STOR,IER)
      PRINT*,'                 EIGENVALUES OF AO + BOG'
      PRINT*,'   IER = ',IER
      DO 9  I=1,NO2
9     PRINT*,'                 ',W1(1)
      PRINT*,' EIGENVALUES OF AC - KCO'
      CALL EIGRF(AKC,NO2,NDIM,0,W1,TEN,NDIM,STOR,IER)
      PRINT*,'   IER = ',IER
       DO 66 I=1,NO2
66    PRINT*,'               ',W1(I)
      IF(DEC.EQ.1) THEN
      PRINT*,'   EIGENVALUES OF A+ BG SYSTEM 2'
      CALL EIGRF(ABG2,NS2,NDIM,0,W1,TEN,NDIM,STOR,IER)
      PRINT*,'   IER= ',IER
      DO 67 I=1,NS2
67    PRINT*,'          ',W1(I)
      IER=
      PRINT*,'   EIGENVALUES OF A - KC SYSTEM 2'
      CALL EIGRF(AKC2,NS2,NDIM, ,W1,TEN,NDIM,   ,IER)
      PRINT*,'   IER= ',IER
      DO 68 I=1,NS2
68    PRINT*,'          ',W1(I)
      ENDIF
      IF(ZZ.EQ.1)GOTO 2
      CALL FORMC(CC,PHIS,NS,NS2,NSEN,IS)
      CALL TFR(CST,CC,NSEN,NS2,1,2)
      DO 1 0 I=1,NS
      DO 1 0 J=1,NSEN
1 0   V(I,J)=CST(I,J)
      CALL LSVDF(V,    ,NS,NSEN,   ,TEN,NDIM,-1,SING,STOR,IER)
      PRINT*,'   LSVDF IER= ',IER
      P= NSEN  -NS
```

```
      IF (P.LT.1) THEN
      DO 101 I=1,NSEN
101   TR(I,1)=V(I,NSEN)
      P=1
      ELSE
      DO 199 I=1,NSEN
      DO 199 J=1,P
199   TR(I,J)=V(I,(J+NS))
      ENDIF
      PRINT*,'   TRANSFORMATION MATRIX'
      CALL FRNT(TR,NSEN   ,P)
      CALL MMUL(CST,TR,NS2,NSEN,P,CSTR)
      PRINT*,'   CST* TR'
      CALL FRNT(CSTR,NS2,P)
      PRINT*,'  THE SINGULAR VALUES'
      CALL FRNT(SING,NS,1)
      CALL TFR(TRT,TR,NSEN,P,1,2)
      CALL MMUL(TRT,TR,P,NSEN,P,RT)
      CALL GMINV(P,P,RT,RT1,J,TAPE)
      CALL FORMC(CC,PHIS,NC,NC2,NSEN,IC)
      CALL TFR(CT,CC,NSEN,NC2,1,2)
      CALL MMUL(TRT,CC,P,NSEN,NC2,TCC)
      CALL MMUL(CT,TR,NC2,NSEN,P,CTT)
      CALL MMUL(CTT,RT1,NC2,P,P,STCR)
      CALL MMUL(STCR,TCC, NC2,P,NC2,STCC)
      ZZ=1
      IF(DEC.EQ.1) THEN
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      DO 7  I=1,NC
      DO 7  J=1,NACT
7     V(I,J) = BC((I+NC),J)
      CALL LSVDF(V,NDIM,NC,NSEN,TEN,NDIM,-1,SING,STO ,IER)
      PRINT*,'   LSVDF FOR CONTROL SPILL OVER IER = ',IER
      E= NACT - NC
      IF(E.LT.1) THEN
      DO 71 I=1,NACT
71    T1(I,1) = V(I,NACT)
      E=1
      ELSE
      DO 72 I=1,NACT
      DO 72 J=1,E
72    T1(I,J) = V(I,(J+NC))
      ENDIF
      PRINT*,'   SYSTEM 2 TRANSFORMATION MATRIX'
      PRINT*,' '
      CALL FRNT(T1,NACT,E)
      PRINT*,'  '
      PRINT*,'  THE SINGULAR VALUES OF B1'
      CALL FRNT(SING,NC,1)
      CALL TFR(TT,T1,NACT,E,1,2)
      CALL MMUL(TT,T1,E,NACT,E,R)
      CALL GMINV(E,E,R,R1,J,TAPE)
      CALL FORMB(BC,PHI,NS,NS2,NACT,IS)
      CALL MMUL(BC,T1,NS2,NACT,E,BCT)
      CALL MMUL( [illegible] )
      CALL MMUL(BSRT,TT,NS2,E,NACT,BST)
      CALL TFR(STCR,BC,NS2,NACT,1,2)
```

```
      CALL MMUL(BST,STOR,NS2,NACT,NS2,BSBT)
      ENDIF
      GOTO 1
20    CONTINUE
      PRINT*,'   THE CHECK FOR LINEAR COMBONATIONS ON CONTROL'
      L=NS+1
      DO 301 I=1,NC
      M=IC(I)
      CALL FORMC(CC,PHIS,NS,NS2,NSEN,IS)
      DO 301 K=1,NS
      DO 301 E=1,NSEN
301   V(E,K) = CC(E,K)
      DO 302 J=1,NSEN
302   V(J,L) = PHIS(M,J)
      CALL LSVDF(V,NDIM,NSEN,L,TEN,NDIM,-1,SING,STOR,IER)
      PRINT*,'   THE CONTROL MODES USED IN THE CHECK'
      PRINT*,(IS(K),K=1,NS),M
      PRINT*,'   SINGULAR VALUES'
300   CALL PRNT(SING,L,1)
      IF (DEC.EQ.1) THEN
      L = NC +1
      DO 303 I=1,NS
      M=IS(I)
      CALL FORMB(BC,PHI,NC,NC2,NACT,IC)
      DO 304 K=1,NC
      DO 304 E=1,NACT
304   V(K,E) = BC((K+NC),E)
      DO 305 J=1,NACT
305   V(L,J) = PHI(M,J)
      CALL LSVDF(V,NDIM,L,NACT,TEN,NDIM,-1,SING,STOR,IER)
      PRINT*,'   THE CONTROL MODES USED IN THE CHECK'
      PRINT*,(IC(K),K=1,NC),M
      PRINT*,'   THE SINGULAR VALUES'
303   CALL PRNT(SING,L,1)
      ENDIF
      PRINT*,'   TO CHANGE THE WEIGHTING MATRIX ENTER 1'
      PRINT*,'   TO REARRANGE MODES FOR N = ',N,' OR TO MAKE A '
      PRINT*,'   DECOUPLED RUN ENTER 2 '
      PRINT*,' TO TERMINATE THE RUN ENTER 3 > '
      READ *,Q
      PRINT*,Q
      IF (Q.EQ.1) THEN
      GOTO 12
      ELSEIF (Q.EQ.2) THEN
      GOTO 299
      ENDIF
      END
```

MAP--(LO=A)

-ADDRESS--BLOCK	------PROPERTIES-------	TYPE	---------SIZE	-NAME	---ADDRESS--BLOCK
5213		REAL		ACT	[illegible]
[illegible]3		REAL	144	AKC	31[illegible]
[illegible]		REAL	144	AKCI	[illegible]
355[illegible]3		REAL	144	BB	346228
161[illegible]3		REAL	144	BC	36363B

```
      SUBROUTINE TIME(EAT2,MM,DT,X1,XO,MODE,EAT,WORK,DEC)
      COMMON/MAINA/NDA,NDA1
      COMMON/SAVE/T(100),TS(100)
      COMMON/NUM/IC(12),IS(12),IR(12),NC,NS,NR
      REAL XO(NDA),EAT2(NDA,NDA),EAT(NDA,NDA),DT,MODE(2,12)
      REAL WORK(NDA,NDA),A,AA,Z,X1(NDA)
      INTEGER MM,DEC,Z7
      N=1
      KK=0
      A=0
 203  CONTINUE
      M=0
 201  CONTINUE
      KK=1
      CALL VMULFF(EAT2,XO,MM,MM,1,NDA,NDA,X1,NDA,IER)
      DO 113 I=1,MM
 113  XO(I) = X1(I)
      M=M+1
      IF((M*DT).EQ.0.5) THEN
      A = A + 0.5
      DO 202 K=1,2
      AA=0.0
      IF(DEC.EQ.0) THEN
      DO 204 I=1,NC
      J=IC(I)
 204  AA= AA+ MODE(K,J) *X1(I)
      DO 205 I=1,NS
      J=IS(I)
 205  AA= AA+ MODE(K,J) *X1(I+NC*4)
      DO 206 I=1,NR
      J=IR(I)
 206  AA= AA+ MODE(K,J) *X1(I+NC*4+NS*2)
      ELSE
      DO 211 I=1,NC
      J=IC(I)
 211  AA = AA + MODE(K,J) * X1(I)
      DO 212 I=1,NS
      J=IS(I)
 212  AA = AA + MODE(K,J) * X1(I + NC*2)
      ENDIF
      T(N)= AA
      N=N+1
 202  CONTINUE
      IF(A.GE.20.0)  GOTO 210
      GOTO 203
      ELSE
      GOTO 201
      ENDIF
 210  CONTINUE
      A= .5
      L=2*(20/A)
      PRINT*,'    TIME              X                    Y'
      DO 1 I=1,L,2
       WRITE(*,9)A,T(I),T(I+1)
 1    A=A+.5
      PRINT*,'   '
 9    FORMAT(3X,F6.1,4X,F15.6,4X,F15.6)
```

```
UBROUTINE FORMA        74/74    OPT=0                              FTN 5.1+528

              SUBROUTINE FORMA(A,D,W,N,N2,IC)
              COMMON/MAIN1/NDIM
              REAL A(NDIM,NDIM),W(12),D(12)
              INTEGER IC(N),I,J,N,M
              DO 1 I=1,N2
              DO 1 J=1,N2
              A(I,J)=0.0
        1     CONTINUE
              DO 2 I=1,N
              M= IC(I)
              A((I+N),(I+N))= D(M)
              A(I,(I+N)) = 1.0
              A((I+N),I) = -(W(M)**2)
        2     CONTINUE
              RETURN
              END
```

```
UBROUTINE FORMC        74/74    OPT=0                              FTN 5.1+528

              SUBROUTINE FORMC(C,PHIS,N,N2,NSEN,IC)
              COMMON/MAIN1/NDIM
              REAL C(NDIM,NDIM),PHIS(NDIM,NDIM)
              INTEGER IC(N),M,NSEN,N,N2,I,J,IP
              DO 1 I=1,NSEN
              DO 1 J=1,N2
              C(I,J)=0.0
        1     CONTINUE
              DO 2 I=1,NSEN
              DO 2 J=1,N
              M=IC(J)
              C(I,J)= PHIS(M,I)
        2     CONTINUE
              END
```

```
UBROUTINE FORMB        74/74    OPT=0                              FTN 5.1+528

              SUBROUTINE FORMB(B,PHI,N,N2,NACT,IC)
              COMMON/MAIN1/NDIM
              REAL B(NDIM,NDIM),PHI(NDIM,NDIM)
              INTEGER IC(N),NACT,N,M,I,J,IP
              DO 1 I=1,N2
              DO 1 J=1,NACT
              B(I,J) = 0.0
        1     CONTINUE
              DO 2 I=1,N
              DO 2 J=1,NACT
              M= IC(I)
              B((N+I),J) = PHI(M,J)
        2     CONTINUE
              END
```

```
      SUBROUTINE FORMX1(XO,INIT)
      COMMON/NUM/IC(12),IS(12),IR(12),NC,NS,NR
      REAL XO(4.),INIT(4,12)
      INTEGER M
      DO 1 I=1,NC
      M=IC(I)
      XO(I) = INIT(1,M)
      XO(I+NC) = INIT(2,M)
      XO(I+2*NC) = INIT(3,M)
 1    XO(I+3*NC) = INIT(4,M)
      DO 2 I=1,NS
      M=IS(I)
      XO(I+4*NC) = INIT(1,M)
      XO(I+4*NC+NS) = INIT(2,M)
      XO(I+4*NC+2*NS) = INIT(3,M)
 2    XO(I+4*NC+3*NS) = INIT(4,M)
      DO 3 I=1,NR
      M=IR(I)
      XO(I+4*NC+4*NS) = INIT(1,M)
 3    XO(I+NC*4+NS*4+NR) = INIT(2,M)
      END
```

```
      SUBROUTINE FORMXO(XO,INIT)
      COMMON/NUM/IC(12),IS(12),IR(12),NC,NS,NR
      REAL XO(4.),INIT(4,12)
      INTEGER M
      DO 1 I=1,NC
      M= IC(I)
      XO(I) = INIT(1,M)
      XO(I+NC) = INIT(2,M)
      XO(I+NC*2)  =  INIT(3,M)
 1    XO(I+NC*3)  =  INIT(4,M)
      DO 2 I=1,NS
      M=IS(I)
      XO(I+NC*4)     =INIT(1,M)
 2    XO(I+NC*4+NS)  =INIT(2,M)
      DO 3 I=1,NR
      M=IR(I)
      XO(I+NC*4+NS*2)     =INIT(1,M)
 3    XO(I+NC*4+NS*2+NR) = INIT(2,M)
      END
```

```
      SUBROUTINE PRNT(MAT,N,M)
      COMMON/MAIN1/NDIM
      REAL MAT(NDIM,NDIM)
      INTEGER N,I,J
      PRINT*,' '
      DO 1 I=1,N
      PRINT'(1X,F12.4)',(MAT(I,J),J=1,M)
 1    CONTINUE
      PRINT'(///)'
      RETURN
      END
```

Vita

William Thomas Miller was born October 3, 1950, in St Louis, Missouri. Graduating in 1969 from St Mary's High School, which is located in St Louis, he received an appointment to the United States Air Force Academy. He graduated in 1973 with a Bachelor of Science in Aeronautical Engineering and a regular commission in the United States Air Force. He attended Undergraduate Pilot Training at Laughlin Air Force Base, Texas. After graduating, he was assigned to the 97th Air Refueling Squadron at Blytheville Air Force Base, Arkansas. He ramined at Blytheville on a combat crew, advancing to aircraft commander until his assignement to the AFIT School of Engineering.

Permanent Address: [illegible] Humphrey Street
St Louis, Missouri 63118

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GAE/AA/81D-20	2. GOVT ACCESSION NO. AD-A111 771	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DECENTRALIZED CONTROL OF LARGE SPACE STRUCTURES		5. TYPE OF REPORT & PERIOD COVERED MS Thesis
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William Thomas Miller, Capt , USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE December 1981
		13. NUMBER OF PAGES 74
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

28 JAN 1982

18. SUPPLEMENTARY NOTES

APPROVED ... 190 17.

Major, USAF

... Public Affairs

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Air Force Institute ...

Wright ...

Decentralized control

Linear system

Modal control

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A development and analysis of a single controller, before and after the elimination of "spillover" terms, is implemented to attempt to achieve desired response characteristics of the structure under evaluation. Using this derived data as a basis for comparison, a pair of decentralized controllers are implemented on the structure. Problems encountered with the implementation of more than two decentralized controllers are investigated. The structure used for the investigation is a lumped mass tetrahedron.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

DATE
FILMED

3-82

DTIC